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A DETERMINATION OF THE PHOTOGRAPHIC MAGNITUDES OF COMPARISON STARS IN CERTAIN OF THE HAGEN FIELDS

By C. H. GINGRICH

To the ancient astronomer the features of the sky, except for the moon, the planets, and the appearance of an occasional comet, were unchangeable. This condition was attributable, not to a lack of diligence on the part of the observers, but to the fact that they did not have at their disposal any of the instruments of great precision which at present are indispensable to the advancement of science in general and of the science of astronomy in particular. However, changes of such a character were taking place as to be sure, sooner or later, to attract the notice even of the unaided eye.

In 1596 David Fabricius saw in the constellation *Cetus* an unusually bright star. When Bayer made the chart of this region seven years later a star in the same position was designated "*o*" (omicron), it being at that time relatively very faint. In 1609 Fabricius again saw the star bright. These facts constituted the evidence that the light received from this particular star was not constant in quantity. It has since been found to vary to such an extent that at certain periods at least a thousand times as much light is received from the star as at other periods. When, approximately a half-century later, the character of this change was understood and the period of the variation was fixed at about eleven months, the name *Mira*,

as an alternative to Bayer's letter, was assigned to the star to indicate that it was a cause of amazement that the light of the star should be variable.

For a long time *Mira Ceti* was the only star known to be variable. During the following two centuries others were noted but the study of variable stars was not systematized until the first half of the nineteenth century, when Argelander turned his attention to this subject and brought it into prominence as a new branch of astronomy. During the latter part of the nineteenth century several catalogues of variable stars and the elements of their variation were published, one by Schönfeld in 1865 containing 113 variable stars, and three consecutive ones by S. C. Chandler, the latest in 1896 containing 393 stars.[1] *A Provisional Catalogue of Variable Stars* published by the Harvard College Observatory in 1903 makes reference to 1227 stars. During the year 1904 more than 400 were discovered at the Harvard Observatory as a result of a systematic examination of the photographic plates having as its object the detection of variables. This search for variable stars is still going on and they are being found in such large numbers that at present the total number of known variables is in the neighborhood of 4000. In his *Katalog und Ephemeriden veränderlicher Sterne für 1912*, Ernst Hartwig gives the elements and predicted times of maximum and minimum for 1912 of 1370 stars. With the number of variables being so rapidly augmented it is unsafe to assume that any particular star is not a variable. This is made the more emphatic by the recent discovery that stars which for a long time were considered as standards because of their seemingly absolute uniformity are also subject to change. Notable among these is *Polaris*, which was announced by Hertzsprung[2] as being subject to a slight variation. The change in brightness was subsequently confirmed by E. S. King[3] and by Stebbins.[4] According to the *H.C.O. Circular 174* the range of variability is about 0.11 magnitude in a period of nearly 4 days.

[1] *Astronomical Journal*, No. 379, 1896.

[2] *Astronomische Nachrichten*, **189**, 89, 1911.

[3] *Science*, **34**, 523, 1911; *Harvard Annals*, **59**, 249.

[4] Paper before the Astronomical and Astrophysical Society of America, December, 1911.

Because of the attention which the subject of variable stars was receiving, Rev. J. G. Hagen in 1890, then the director of the Georgetown College Observatory, Washington, D.C., now of the Specola Vaticana, Rome, undertook the preparation of an atlas which should be useful for identifying the variables and supplying sequences of comparison stars, thus facilitating the work of observing the variables. This *Atlas of Variable Stars* consists of 311 charts and a catalogue sheet for each. The charts are arranged in six series according to position in the sky and according to the brightness of the respective variables at maximum and at minimum, the sixth being a supplement. The charts of the fainter variables cover a region one degree square having the variable in the center. In this square all of the *B.D.* stars are plotted and later verified as to position provisionally by means of a five-inch equatorial and finally by means of the twelve-inch equatorial of the Georgetown College Observatory. In the central region one-half degree square are plotted, in addition to the *B.D.* stars, all the stars visible in the twelve-inch telescope. The positions of all the stars were determined by means of a glass reticle and a chronograph, as described in the prefaces of Series I, II, and III.

Not only the positions but also the relative magnitudes of the stars in these fields are given in this atlas. The stars are connected by sequences of brightness according to the method of Herschel and Argelander. The grades were changed by the linear formula,

$$\text{Magnitude} = a + b(\text{Grade} - C),$$

into magnitudes so that they should agree as nearly as possible with the *B.D.* scale between the seventh and the tenth magnitudes. In some cases stars as faint as the 13.5 magnitude on the linear scale adopted are plotted. For the fainter stars the indicated magnitudes are merely relative and are not meant to be a continuation of either the *B.D.* or the Harvard scale. The stars, however, serve the purpose of the practical observer, since they furnish the successive comparison stars for the variable as it changes its light-intensity, and, although they do not enable one to establish the magnitude of the variable at minimum, they furnish data for determining the magnitude in any photometric scale by means of the

sequences of grades. For one who desires to make visual observations of the variables these charts, together with the catalogue sheets which accompany them, furnish the necessary equipment as far as the identification of the variable and the choice of suitable comparison stars are concerned.

In recent years photographic methods have invaded nearly every domain of astronomical research so that now there is scarcely any branch of the science which cannot be pursued more successfully by use of the photographic plate than visually. The relative merits of the two methods are rather obvious and have been treated elsewhere, so they will not be discussed here. The investigations of the light-changes in variable stars and the determination of their light-curves and hence their periods are facilitated very greatly by photographic processes. In order to study the variables photographically, however, it is necessary to have at hand data corresponding to those which the Hagen *Atlas* furnishes for the visual work. For the identification of the stars in question the charts still serve, but, since in general there is a difference between the visual and the photographic magnitude of a star, the visual determinations do not furnish photographic standards. At the suggestion of Professor J. A. Parkhurst about two years ago, it was thought to be desirable to establish the photographic magnitudes of stars which might be used as standards for comparison in the Hagen fields. It was with this object in view that the present work was begun, although up to the present time it has been possible to accomplish the end for a comparatively small number of the fields in the Hagen list.

The reason for the difference between the so-called photographic magnitudes and the visual magnitudes is inherent in the ordinary photographic plate. By the use of the properly chosen color-filter and a plate that is specially sensitized, the plate can be made to give visual results. However, this feature will be referred to later, and it will be seen that the use of the ordinary plate without a filter is more practicable. In ordinary commercial use of the photographic plates the speed of the plate is not such an important feature, since the operator can usually bring about conditions of light so that the plate will be affected sufficiently in the desired

time. In stellar photography, however, it is impossible to exercise any control upon the source of light, and in case of the fainter stars the quantity of light is so small that the exposure time must be correspondingly long. There is obviously a natural limit to the length of an exposure, since it is not advisable for quantitative work to resume an exposure after the intervention of a day. Consequently a necessary qualification of the plate to be used in stellar ph tographic work is that it must be rapid. The ordinary photographic plates which meet the condition are not equally sensitive to the light in the different parts of the spectrum, but have their greatest sensitiveness in the blue region. It follows directly that the light-effect of a given star is dependent upon its color as well as upon its intensity. Fig. 1 shows that the maximum sensitiveness of the Seed plate, the kind that was used in this work, lies well toward the violet end of the visible spectrum in the neighborhood of λ 4300. On the other hand, it is a physiological fact that the normal eye has its maximum sensitiveness to the light of the yellow-green part of the spectrum. From these considerations it is seen that the scale of photographic magnitudes and the scale of visual magnitudes will necessarily diverge, and that the difference between them in a given case will depend upon the color of the star in question. But the color of the star is merely an indication of more essential characteristics, namely, its chemical constitution and temperature. These conditions are portrayed in the spectrum of he star. The three factors—the photographic magnitude, the visual magnitude, and the spectrum—which pertain to a star are seen to be related, and if the relation is known it is possible, having given any two, to determine the third. This relation has been

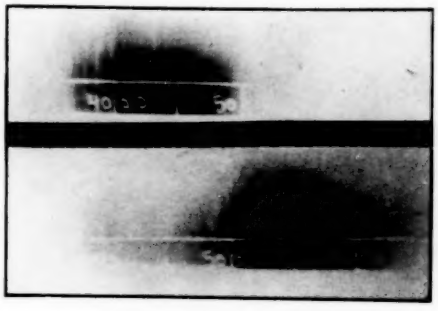

FIG. 1

(*a*) Region of maximum sensitiveness of Seed plates.

(*b*) Region of maximum sensitiveness of Cramer trichromatic plates with color-filter.

carefully studied by King,[1] by Parkhurst,[2] and by Schwarzschild.[3] The determination of photographic magnitudes therefore is not a separate and unrelated branch but is intimately connected with the determination of visual magnitudes and spectral type.

The plan of the work was necessarily conditioned upon, if not entirely determined by, the equipment which was at hand. The instruments which were best adapted to the present undertaking were the six-inch Zeiss ultra-violet camera,[4] and the two-foot reflector.[5] The camera was used for the brighter stars. Because of the short focus, it afforded a large usable field, thus making it possible to find standards, upon which to base the subsequent magnitudes, on the same plate with the variable. This instrument made it possible to use stars which were situated three degrees from the axis with a maximum correction for the distance from the axis of 0.32 magnitude. The usable field of the reflector being very much smaller, it was necessary to establish new standards on the camera plates, which should be near enough to the variable to be used as standards on the reflector plates. The plan, therefore, was to take a series of plates with the camera with proper exposure to determine magnitudes of stars near the variable. The original plan was to make three series of exposures with the reflector with apertures 12, 18, and 24 inches, respectively, for the purpose of passing constantly to fainter stars and necessarily extending the measures over smaller fields. It was found, however, that, owing to the uncertainty in focusing closely enough and because of the distortion of the field when the full aperture was used, it was inadvisable to use more than 12 inches aperture. The reduction from 24 inches to 12 inches aperture reduces the light-effect in the ratio of 5 to 1 approximately, hence the exposure time was greatly lengthened, although not in the same ratio; with the result that it was not possible to measure as faint stars as would have been possible if the original plan had been feasible.

[1] *Harvard Annals*, **59**, 180.

[2] *Astrophysical Journal*, **27**, 169, 1908; **37**, 217, 1912.

[3] *Aktinometrie*, B, p. 19.

[4] *Astrophysical Journal*, **36**, 171, 1912.

[5] *The Study of Stellar Evolution*, Hale, p. 43.

At the outset it was necessary to have stars whose magnitudes were known so that they might serve as a foundation upon which to build. These standard stars were selected according to the condition that the photographic magnitude and the visual magnitude of stars between 5.5 and 6.5 whose spectrum is of the first type, or of class Ao, shall be the same. This definition of the starting-point of photographic magnitudes was adopted by the Committee on Magnitudes of the Astrographic Chart,[1] and it has now become essentially the "International System." This plan then made it possible to use the visual magnitudes of the white stars as determined by Müller and Kempf (*Potsdam Publications*, **17**), in which are given their visual determination of all the *B.D.* stars of 7.5 magnitude or brighter. From this large list it was possible to find standards for the camera plates in all the fields that were undertaken. The plan adopted applied strictly only to white stars, that is, to stars of class Ao. In order to determine in each case which of the bright stars were of this class an objective-prism plate[2] of each field was taken. In some cases it was not possible to find a sufficient number of stars in class Ao, but in only one case were stars used which were farther advanced than A5. In all cases corrections were made for the deviation from class Ao, according to a spectrum correction determined by Parkhurst. During the progress of this work a new determination of the spectrum correction[3] extending to stars of class M was made by Parkhurst, which so accurately accounts for the color-index that any of the Müller and Kempf stars might now be used with the same assurance as stars of class Ao, provided only that the spectral type is known.

The camera plates were taken inside the focus[4] according to the methods adopted by Parkhurst and Jordan. The star images on the plates are circular and all essentially of the same size, about 1.2 mm in diameter. The difference in the light-intensity is indicated by the difference in opacity of the images and not by the size. In addition to exposing the plate to the sky, a series of

[1] *Astronomische Nachrichten*, **186**, 40, 1910.

[2] *Popular Astronomy*, **19**, 595, 1911.

[3] *Astrophysical Journal*, **36**, 217, 1912.

[4] *Ibid.*, **36**, 171, 1912.

twenty images resembling the star images was put upon the plate with a Spurge sensitometer in the laboratory before development.[1] The artificial stars were also of different densities and the steps between them known. All of these images were measured in the Hartmann "Mikrophotometer,"[2] and from the scale-readings and the known ratio of light-intensity of the artificial images the curve was made to which to refer the standard stars and those whose magnitudes were desired on that plate. From the first to the last of the artificial stars there is a gradual increase in opacity. The range is somewhat greater than can be measured with the wedge in the "Mikrophotometer," so that if the first are measurable the last are too dense, and if the last are measurable the first are too faint. Since, as would naturally be expected, there is a similarity in the form of the reduction-curve from the different plates, some of the plates were reduced by using the mean curve from a number of plates. In most cases, however, the curve was drawn for the individual plate from the measures of the images upon it. During the progress of this work an arrangement was devised for rotating the sensitometer during the laboratory exposure to avoid any illusory results in the sensitometer images which would arise if the plate containing the openings were not uniformly illuminated. Fig. 2 shows the curve which was made from the measures on plate UV 987. From this curve, which is typical of all the curves,[3] it may be seen that, except in the upper and lower parts of the curve, very accurate differences in magnitudes are indicated, since differ-

[1] *Astrophysical Journal*, **26**, 244, 1907.

[2] *Ibid.*, **36**, 171, 1912.

[3] During the time which intervened between the preparation and the publication of this paper, Professor Parkhurst has been studying the effect upon the particular brand of plates used in this work due to a variation in the exposure time. The results of the investigation up to the present seem to indicate that the gradation of the plate varies with the exposure time, in such a way that the contrast increases with the exposure. This may later be found to necessitate a slight correction to the magnitudes derived from the extra-focal plates, since the star images on these plates, exposed from 30 to 120 minutes, are compared with the sensitometer images, exposed a few seconds. It may also modify the results obtained from focal measures, since the value of b used in the reductions was also derived from exposures much shorter than the star images. This correction, if needed at all, will be relatively a small one, and may be applied as a percentage correction, starting from the magnitudes of the standard stars for the extra-focal plates, as given in the catalogue sheets.

ences in opacity corresponding to 0.1 or 0.2 mm in the scale-reading could easily be detected in measuring.

Two corrections were applied to reduce each star to the center of the plate, the plate having been placed in the camera so that its center was in the axis of the lens. The one correction was made

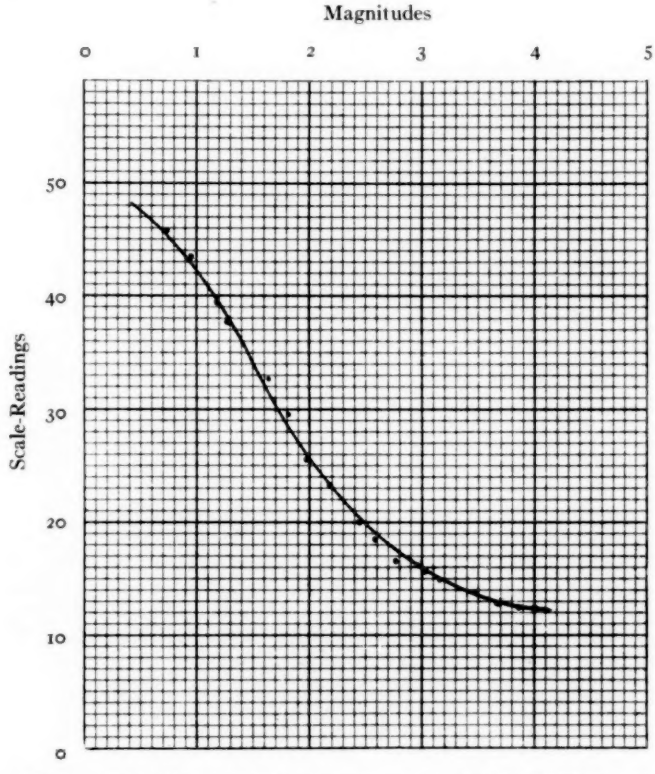

FIG. 2.—Reduction curve from measures of extra-focal images, made with the Hartmann "Mikrophotometer."

with reference to ρ, the distance from the axis, and the other with reference to the differential zenith distance. The correction for ρ was determined in the more or less familiar way of making a series of exposures of the same duration on some bright star, usually *Polaris*, the camera being moved in declination between the exposures in order to bring the images at varying distances from the

axis. The time consumed in making such a series did not exceed eight minutes at most, so that it could safely be assumed that on a clear night there would be no noticeable change in transparency in that short time. Nor would any slight variation in the star such as was mentioned in the case of *Polaris* have any appreciable effect in that period. Hence any difference in the various images would be due to the difference in the distances from the axis. It is reasonable that such correction should be important because, when the plate is placed inside the focus, the edges of the plate are of necessity at a less distance from the focus than the center of the plate. This correction was determined by Parkhurst and verified by the writer by remeasurements and also by subsequent plates. The corrections have been used as follows:

TABLE I

ρ°	Δ Mag.	ρ°	Δ Mag.
0.0	0.00	1.6	0.09
.2	.00	1.8	.11
.4	.00	2.0	.14
.5	.01	2.2	.17
.6	.01	2.4	.20
0.8	.02	2.6	.24
1.0	.03	2.8	.28
1.2	.05	3.0	.32
1.4	0.07	3.2	0.36

The correction for differential zenith distance, that is for atmospheric absorption, is much smaller and is determined from the table given by Wirtz.[1] This correction has been small in all this work because the exposures were made as near the zenith as possible. It is, however, a systematic correction and needs to be applied.

The following form shows the method of reduction. The scale-readings of the sensitometer images are given from which the curve of Fig. 2 was drawn. The scale-readings of the film near the images measured are made for the purpose of detecting any marked variation in the opacity which sometimes occurs. The columns need no explanation except possibly $(O-s)_2$, in which O stands for the center of the plate and s for the star, the difference being taken with reference to the zenith distance. The column "Sp. Cor." is the

[1] *Astronomische Nachrichten*, **154**, 361, 1901.

spectrum correction, the stars not being purely of class Ao. The column "*M*o" represents the quantity to be added to the magnitudes of the plate in order to bring them to the scale of the adopted standards.

TABLE II

REDUCTION SHEET FOR CAMERA EXTRA-FOCAL PLATE

UV 987 *U Cancri* $\alpha = 8^h30^m$ $\delta = 19°.2$ 1912 April 10 Seed 30

C.S.T. $= 8^h.4$ Sid. T. $= 9^h.7$ H.A. $= 1^h.2$W $z = 26°.0$ $q = 36°$ $\frac{\Delta M}{\Delta z} = 0.006$

ARTIFICIAL STAR IMAGES

No.	Scale-Reading Film	Scale-Reading Image	No.	Scale-Reading Film	Scale-Reading Image
1.....	11.5	12.4	11.....	10.8	25.5
2.....	11.3	12.3	12.....	10.8	29.5
3.....	11.2	12.8	13.....	11.1	32.6
4.....	11.1	13.7	14.....	10.9	37.8
5.....	11.3	14.9	15.....	11.0	39.4
6.....	11.0	15.6	16.....	11.0	43.5
7.....	11.0	16.5	17.....	11.1	45.8±
8.....	10.8	18.4	18.....	....	....
9.....	11.1	19.9	19.....	....	....
10.....	11.0	23.2	20.....	....	....

B.D. Stars	Scale-Reading Film	Scale-Reading Image	Δ Mag. from Curve	ρ	$(O-s)_z$	Cor. ρ	Cor. z	Corr'd ΔMag.	P.D. Mags.	Sp. Cor.	*M*°
20°2178.....	11.3	37.6	1.33	1°.4	+1°.2	0.07	+0.01	1.41	6.92	+0.14	5.65
20°2172.....	11.3	33.8	1.53	0.6	+0.7	0.01	0.00	1.54	7.07	+0.23	5.76
19°2053.....	11.4	34.7	1.48	0.7	−0.3	0.01	0.00	1.49	6.94	+0.19	5.64
Mean...	.....	.....	.....	....	.....	.....	.....	.....	.....	.....	5.68
Hagen Stars									Mags.		
3.....	11.3	14.5	3.13	0.9	−0.8	0.03	0.00	3.15	8.83	.....	....
4.....	11.4	14.4	3.16	0.8	−0.7	0.02	0.00	3.18	8.86	.....	....
6.....	11.3	12.2	4.08	1.0	−0.8	0.03	0.00	4.11	9.79	.....	....

With the standards determined from the infra-focal images it was possible to pass to fainter stars by means of plates taken in focus on the two-foot reflector. In order to secure as large a field as possible, and in order not to overexpose the brighter stars, a series of short exposures with diaphragm giving 12 inches aperture was made. Afterward, by using magnitudes determined from this

series, the magnitudes of still fainter stars were determined from a series of plates having an exposure of at least an hour. A few exposures were made with an aperture of 18 inches, but for reasons mentioned above it was better to use only the 12-inch aperture and nearly all the plates that contribute to the results were taken with that aperture.

The focal images were then measured under the Zeiss-Romare machine.[1] The machine being equipped with two screws, both the vertical and horizontal diameters of the images were measured and the mean adopted. It is very difficult to obtain plates on which the images are perfectly circular. Any deviation from perfect roundness appears in the magnified image which was measured. Only in exceptional cases, however, will the discrepancy amount to more than 15 per cent of the mean, and in most cases it was much less. The reduction to magnitudes was made by the formula

$$\text{Magnitude} = a - b\sqrt{D},$$

in which b is a constant depending upon the instrument and the kind of plate used, and a is a constant which depends upon the exposure. The justification for the use of this formula for the combination of instrument and plate used is found in an investigation by Parkhurst[2] made some time ago.

The value of the constant b was determined from a number of plates by two independent methods. The first method was based upon measures of plates of the *Pleiades*. The images of a list of the stars whose photographic magnitudes were known were measured. By plotting as abscissas the values of $\sqrt{D}$ derived from the plate and as ordinates the magnitudes determined by Schwarzschild on the absolute scale, using the largest range that could be derived from the plate, it was possible to determine the increment in magnitude corresponding to a given increment in $\sqrt{D}$. From the formula, we have

$$M = a - b\sqrt{D}$$

$$M + \Delta M = a - b(\sqrt{D} - \Delta\sqrt{D}).$$

[1] *Astrophysical Journal*, **36**, 175, 1912.

[2] *Ibid.*, **31**, 21, 1910.

By subtraction,

$$\Delta M = b\Delta\sqrt{D}$$

$$b = \frac{\Delta M}{\Delta\sqrt{D}}.$$

Theoretically it would be necessary to use only two stars to determine b in this equation, but in practice as many as were available were used. A straight line was found to represent the mean of all the plotted points, thus justifying the form of the equation. From 14 series of *Pleiades* images the mean value of b was found to be 0.897 with a probable error of ± 0.004. The second method of determining b was the application of a device used by Wirtz[1] and by Schwarzschild[2] and by others. The essential feature of the method is the interposition of a medium which decreases the light of the star by a known amount. This medium must be such as not to have any selective absorption, that is, it must reduce the light of all wave-lengths equally. A fine wire cloth or gitter with uniform meshes meets the requirement. In practice it was used in this work by fixing a piece of the gitter in a metal frame which was placed before the plate so as to bring the gitter at a distance of about 75 mm from the film. This distance was determined such as to prevent the overlapping of the diffraction images and the central image, which precaution was necessary in order to keep the central image free for measurement. From measurements of the constants of the gitter made by Parkhurst, Dr. Van Maanen, and the writer, the following values for the absorption in magnitudes were derived, respectively, 1.892, 1.877, and 1.864. The absorption of the gitter was further tested by Parkhurst with the Hartmann wedge photometer and was found to be 1.86 magnitudes. The value 1.87 was adopted in this work. The gitter used was large enough to cover only half the plate so that when exposed with the gitter half of the plate received free images and the other half received the gitter images. The method of using the gitter was as follows: first, to make an exposure having the gitter covering one-half of the plate and then, after reversing the gitter so as to cover the other

[1] *Astronomische Nachrichten*, **154**, 317, 1901.

[2] *Ibid.*, **183**, 297, 1910.

half, to make a second exposure of the same length as nearly as possible. An objection to the use of the half-gitter, pointed out by Schwarzschild, is the fact that near the middle of the plate there will be images part of whose rays were affected by the gitter but the rest of which were not so affected. These images are necessarily useless in the determination of b. These useless images fall in the region which is normally the best part of the field, which becomes a serious objection when the field is small at best, as in the case of the reflector. The difficulty would be removed by using a gitter over the entire plate during one exposure, and leaving the plate entirely free during the other. In this case it would be assumed that the conditions of seeing remain unchanged during the two exposures, which is also objectionable. In the former case the difficulty is definite and can be eliminated, but in the latter the error introduced is uncertain. Of the two, the former therefore seems to be the less objectionable. Usually in practice an entirely free exposure for a shorter time was made on the same plate and used according to the first method, thus furnishing a direct comparison between the two methods here used for determining the value of b. The free and the gitter images were measured and used for the determination of b as follows:[1]

$$M_1 = a_1 - b\sqrt{D_{1f}} \tag{1}$$

$$M_1 + 1.87 = a_2 - b\sqrt{D_{2g}} \tag{2}$$

$$M_2 = a_2 - b\sqrt{D_{2f}} \tag{3}$$

$$M_2 + 1.87 = a_1 - b\sqrt{D_{1g}} \tag{4}$$

(2)−(1)

$$1.87 = a_2 - a_1 + b(\sqrt{D_{1f}} - \sqrt{D_{2g}}) \tag{5}$$

(4)−(3)

$$1.87 = a_1 - a_2 + b(\sqrt{D_{2f}} - \sqrt{D_{1g}}) \tag{6}$$

Adding (5) and (6)

$$1.87 = \frac{b}{2}(\sqrt{D_{1f}} - \sqrt{D_{2g}} + \sqrt{D_{2f}} - \sqrt{D_{1g}}),$$

[1] M_1 = magnitude of star whose image is free in first exposure.
a_1 = value of a from first exposure.
D_{1f} = diameter of free images in first exposure.
D_{2g} = diameter of gitter image in second exposure.

from which b is readily found. From three plates which were investigated by this method the mean value of b was determined to be 0.914 with a probable error of ±0.01. Since this method is independent of the actual magnitudes of the stars in question, and since the value of b agrees so closely with the value of b as determined from the magnitudes of the *Pleiades*, it furnishes evidence

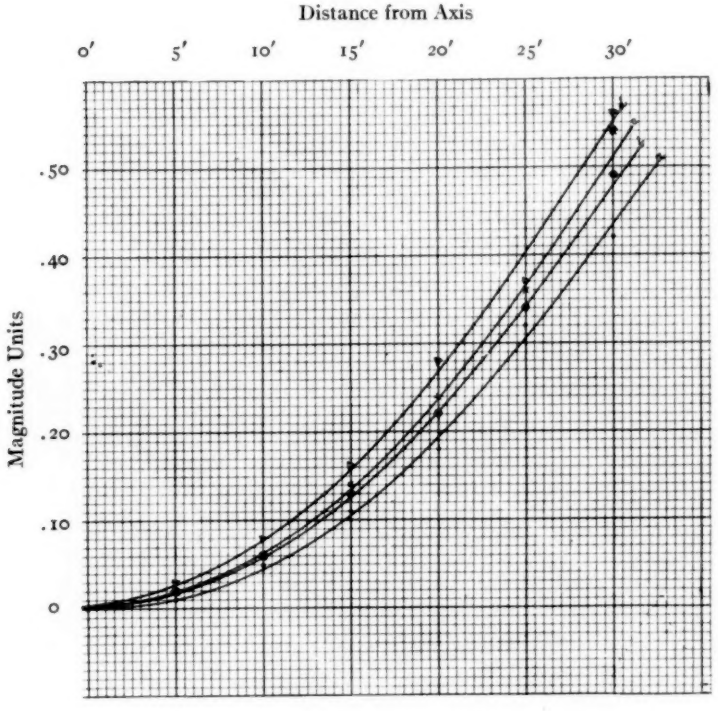

FIG. 3.—Correction curves for distance from axis on reflector plates

that the derived value of b used in the formula mentioned above leads to the determination of magnitudes on the absolute scale upon which the *Pleiades* magnitudes were based. The exposures in the latter case were also made on the *Pleiades*, although that was not necessary as any other group would serve as well, provided only that a suitable number of stars, preferably of the same spectral type, not differing too widely in magnitude, should fall within the

measurable field of the plate. From a combination of the values determined by the two methods, giving equal weight to each individual determination, the value 0.90 was adopted as the value of b to be used.[1]

In the case of the reflector plates the correction for the position of the star with reference to the axis is important. Twenty series of exposures were made for the purpose of determining this correction for the 12-inch aperture. A star near the pole was selected for this purpose so that it would be unnecessary to guide during the exposures. The star chosen was of such a brightness that the exposure time necessary was long enough so that the slight differences of exposures, due to the opening and closing of the slide, would be proportionately unappreciable, and of such a brightness also that the exposure time was not so long as to require for the series so much time that the conditions of seeing should change. A row of twenty images was made by moving the telescope in declination between the exposures, the telescope having been set so that the star should at some time in the series be in the axis. The correction was seen to depend upon the diameter of the image, so the series were made of images of different sizes. The curves in Fig. 3 show the mean values of the corrections for the different sizes of the images. From these curves it is seen that the correction becomes excessive for stars more than 30 minutes of arc from the axis, and for that reason stars so situated were not used. In some cases the stars whose magnitudes are given are more than 30 minutes of arc from the variable but they have been determined from plates so centered that the variable was not in the axis.

The following table of corrections was derived from the curves of Fig. 3, and was used for correcting the plates of 12 inches aperture. The first horizontal row indicates the size of the images; ρ indicates the distance from the axis in minutes of arc. The corrections are given in hundredths of a magnitude.

A similar correction was made for the plates taken with 18 inches aperture. On these plates stars more than 20 minutes of arc from the axis could not be used. The corrections in Table IV were found as the mean of sixteen series. These apply to images for which the value of $b\sqrt{D}$ is equal to 10.0. Since very few plates

[1] See footnote, p. 216.

TABLE III

TABLE OF CORRECTIONS FOR DISTANCES FROM AXIS

REFLECTOR FIELD. SEED PLATES. 12 INCHES APERTURE

$b\sqrt{D}$ / ρ'	7.75	8.00	8.50	9.00	9.50	10.00	10.50	11.50
1′	00	00	00	00	00	00	01	01
2	00	00	01	01	01	01	01	02
3	01	01	01	01	01	02	02	03
4	01	01	01	01	02	02	02	03
5	01	01	02	02	02	02	03	04
6	02	02	03	03	03	03	04	05
7	02	02	03	03	04	04	05	06
8	03	03	04	04	05	05	06	07
9	04	04	05	05	06	06	07	08
10	04	05	06	06	07	07	08	09
11	06	06	07	07	08	09	10	11
12	07	07	08	08	09	11	11	12
13	08	09	10	10	10	12	13	14
14	09	10	11	11	12	13	14	16
15	11	12	13	13	14	15	16	18
16	12	13	14	15	16	17	18	20
17	14	15	16	17	18	19	20	22
18	15	16	18	19	20	21	22	24
19	16	18	20	21	22	24	25	27
20	18	20	22	23	24	26	27	29
21	20	20	24	26	27	28	29	31
22	22	24	26	28	29	30	31	33
23	24	26	28	30	31	32	33	35
24	26	28	31	32	34	35	36	37
25	29	31	34	35	36	37	39	40
26	32	34	36	38	39	40	42	44
27	34	37	40	41	42	43	45	48
28	38	40	43	44	45	46	48	51
29	41	43	46	47	48	49	51	53
30	44	46	49	50	52	53	54	58

TABLE IV

TABLE OF CORRECTIONS FOR DISTANCES FROM AXIS

REFLECTOR FIELD. SEED PLATES. 18 INCHES APERTURE

ρ'	Mag.	ρ'	Mag.
1	0.01	11	0.14
2	.01	12	.17
3	.02	13	.20
4	.03	14	.24
5	.04	15	.27
6	.05	16	.31
7	.07	17	.34
8	.08	18	.38
9	.10	19	.42
10	0.12	20	0.46

made with this aperture were used, the corrections for images of other sizes were not determined.

Since the usable field of the reflector never exceeds 1° in diameter and the plates in general were taken very near the meridian, none of the fields being south of 10 degrees north declination, the correction for differential atmospheric absorption was negligible.

The following form (Table V) is an example of the measurements and complete reduction of some of the stars on one of the reflector plates.

The columns marked x and y are respectively the horizontal and vertical diameters. Each of the numbers recorded represents the mean of three settings of the micrometer. The column "Mean D" is expressed in half-revolutions of the micrometer screw. The column "μ" is obtained by multiplying the preceding column by the factor 36.7 which converts the half-revolution units into thousandths of a millimeter.

TABLE V

REDUCTION SHEET FOR REFLECTOR PLATE

R2657 χ *Cygni* 1912 June 15, 11^h58^m to 12^h23^m C.S.T. 12 In. Apert.
Seed 30

Hagen No.	x	D_x	y	D_y	Mean D	μ	$\sqrt{D}$	$.90\sqrt{D}$	ρ	Corr'n ρ	Corr'd $b\sqrt{D}$	Mags. (C)	a
3 ...	46.45		44.35										
	51.84	5.39	49.84	5.49	5.44	199.65	14.13	12.72	24	39	12.33	6.92	19.25
9 ...	47.98		46.70										
	52.25	4.27	50.55	3.85	4.06	149.00	12.21	10.99	14	15	10.84	8.79	19.63
7 ...	47.41		45.68										
	51.80	4.39	49.93	4.25	4.32	158.54	12.59	11.33	25	40	10.93	8.43	19.36
13 ...	47.76		46.86										
	51.65	3.89	50.34	3.48	3.68	135.06	11.62	10.46	24	36	10.10	9.23	19.33
													19.39
												Mags. (R)	
20 ...	48.60		47.45										
	52.25	3.65	50.81	3.36	3.51	128.82	11.35	10.22	06	03	10.19	9.20	.....
34 ...	48.51												
	51.63	3.12	50.61	2.82	2.97	109.00	10.44	9.40	08	05	9.35	10.04	.....
27 ...	48.22		47.53										
	50.65	2.43	49.75	2.22	2.33	85.51	9.25	8.32	10	06	8.26	11.13	.....
6 ...	47.85		47.12										
	51.90	4.05	50.70	3.58	3.82	140.19	11.84	10.66	16	18	10.48	8.91	.....
26 ...	48.04		46.90										
	51.64	3.60	50.11	3.21	3.40	124.78	11.17	10.05	22	30	9.75	9.64	.....

The column "ρ" expresses the distance from the axis in minutes of arc.

The column "Mags. (C)" indicates magnitudes derived from the camera plates. These magnitudes are used as standards on this plate.

The column "Mags. (R)" indicates the magnitudes derived from this plate.

In addition to the determination of photographic standards from the Seed plates taken with the Zeiss camera, a second series of plates was taken for the purpose of establishing visual standards. The method for accomplishing this end has been used at this observatory for some years.[1] It consists in placing immediately in front of the film of the color-sensitive plate a "visual luminosity" filter which was prepared especially for this purpose by R. J. Wallace. With this filter are used Cramer trichromatic plates. This combination is found by spectroscopic tests to have its maximum sensitiveness in the same region as the normal eye (Fig. 1), and for that reason leads directly to visual magnitudes, although they are determined photographically. These plates were taken in focus and measured on the same machine as the reflector plates. The formula

$$\text{Magnitude} = a - b\sqrt{D}$$

was used in this case also for reducing to magnitudes. The value of b was determined by the method of *Pleiades* magnitudes only. From sixteen series of measures the weighted mean of the value of b was found to be 0.515.

The stars used as standards on these plates were the same as those used for the photographic magnitudes. In this case, however, no correction was applied for the spectrum inasmuch as the question here was solely one of visual magnitudes. The stars whose magnitudes were derived on these plates were the same as those determined from the infra-focal plates taken with the camera. The original intention was to use these visual standards on a series of reflector plates made also with the color-filter. This plan has up to the present time not been carried out for the reason that the exposure times were necessarily very much lengthened because of

[1] *Astrophysical Journal*, **27**, 169, 1908.

the interposition of the color-filter. It was found that normally the trichromatic plates with the filter required nine times the exposure that was required by the ordinary plate. Unfortunately when the trichromatic plates were to be used the emulsions then obtainable were only about half as rapid as the normal emulsion, and consequently it would have been necessary to prolong the exposure time so much as to make it inadvisable to attempt the work. It is hoped that at some time conditions will be found more favorable for carrying out this plan. In the reductions corrections were applied for distance from the axis and for the atmospheric absorption due to the differential zenith distance. The former correction was determined by a method similar to that used for determining the corresponding correction for the infra-focal plates and for the reflector plates. As in the case of the reflector plates, it was found to depend upon the size of the image. The second correction is very small for the reason mentioned in the case of the photographic magnitudes, namely, that the plates were taken near the meridian, and, since all the fields cross the meridian at a considerable altitude, the extinction factor there is very small. This correction is made according to the visual extinction table given in the *Potsdam Publications*, **3**, 285.

In the catalogue which follows (Tables VI–XV), in each field the first section of the table gives the list of standard stars used. The number under the Potsdam *Durchmusterung* refers to the serial number in *Publikationen des Astrophysikalischen Observatoriums*, Potsdam, **17**. The color given is the visual estimate by the observers. The other columns need no further comment.

The second section gives the photographic magnitudes determined in this work. In the first column are given the numbers assigned to these stars in the Hagen *Atlas*. The second column gives the visual magnitude assigned by Hagen. They are given here for convenient comparison with the photographic magnitudes. The differences presumably are largely due to spectral type. A part may be due to different scales. In the third column are the weighted mean values determined from the photographic plates after applying a correction of −0.28 magnitude. It was found by Parkhurst that the mean correction necessary to reduce the Pots-

dam magnitudes to the International System was −0.27 magnitude. From a similar determination by Schwarzschild the correction was found to be −0.29 magnitude. Since the magnitudes here

TABLE VI

U CANCRI FIELD

$\alpha = 8^h 30^m 3^s$ $\delta = +19°14'.4$ (1900) Hagen Series II

STANDARD STARS

B.D.		*P.D.*			Spectrum	Cor. for Spectrum	Photographic Magnitude
No.	Mag.	No.	Mag.	Color			
20°2178...	7.0	5204	6.92	GW	A3	0.14	7.06
20 2172...	7.1	5199	7.07	GW	A5	0.23	7.30
19 2053...	7.2	5166	6.94	GW	A4	0.19	7.13

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
2...	7.9	7.70	R	1	....	20...	9.9	11.17	R	1	....
3...	8.5	8.64	C	3	0.04	21...	10.0	11.47	R	1	....
4...	8.5	8.63	C	3	0.02	22...	10.0	10.87	R	1	....
5...	8.7	9.3	R	1	....	23...	10.0	10.78	R	1	....
6...	8.9	8.56	C	3	0.02	24...	10.1	10.71	R	1	....
7...	9.0	9.70	R	1	....	25...	10.3	11.03	R	1	....
9...	9.2	10.15	R	1	....	26...	10.4	11.20	R	4	0.10
10...	9.2	10.42	R	1	....	27...	10.4	11.26	R	1	....
11...	9.3	10.08	R	1	....	28...	10.5	11.84	R	1	....
12...	9.3	10.22	R	4	0.06	29...	10.5	11.19	R	1	....
13...	9.4	9.47	R	1	....	30...	10.8	10.97	R	4	0.08
14...	9.5	10.92	R	1	....	31...	11.0	11.78	R	4	0.10
15...	9.6	10.84	R	1	....	32...	11.0	11.93	R	1	....
16...	9.7	10.63	R	1	....	33...	11.1	11.55	R	4	0.11
17...	9.7	10.92	R	1	....	34...	11.1	11.98	R	4	0.10
18...	9.8	10.96	R	1	....	35...	11.2	11.91	R	4	0.11
19...	9.9	11.66	R	1	....	36...	11.8	12.40	R	4	0.09
						39...	12.3	12.43	R	4	0.08

VISUAL MAGNITUDES

Hagen No.	Hagen Mag.	Magnitude	Color-Index	Spectrum Inferred	Instr.	No. of Plates	p.e.
3.....	8.5	8.37	+0.27	A7	C	3	0.04
4.....	8.5	8.38	+0.25	A6	C	3	0.03

given were based upon Potsdam magnitudes as standards they are subject to the same correction. The mean correction, −0.28 magnitude, was adopted. In the column "Instr." C indicates the Zeiss camera, and R indicates the two-foot reflector. The column p.e. is designed merely to indicate in a general way the agreement of the plates. The number given is found by taking twice the greatest range on the plates and dividing by three times the number of plates. This is a close approximation to the usual probable error formula, which, however, presupposes a larger number of independent determinations than are used here. When the star has been measured on less than three plates, no probable error is given. When, as happens in a few cases, the star was measured on only one plate, the magnitude is given to the nearest tenth. The third section in the fields in which it appears, gives the visual magnitudes determined. In this section two additional columns are given. The column headed "Color-Index" gives the difference between the photographic and visual magnitudes. From this column by reference to Parkhurst's determination of the relation of color-index to spectral type, the next column, "Spectrum Inferred," is formed.

More than half of the stars of this field were measured on only one plate. However, the results given are entitled to more weight than would seem to be the case from a single plate, since a systematic correction was found from the stars that appear on the four plates and applied to the stars that appear on the one plate only so

TABLE VII

R CAMELOPARDALIS FIELD

$\alpha = 14^h25^m6^s$ $\delta = +84°17'.1$ (1900) Hagen Series III

STANDARD STARS

B.D.		*P.D.*			Spectrum	Cor. for Spectrum	Photographic Magnitude
No.	Mag.	No.	Mag.	Color			
81°495....	6.8	7985	7.40	GW−	A1	0.04	7.44
83 431....	6.0	8003	5.87	GW+	F8	0.77	6.64
84 335....	7.5	8043	6.98	WG+	K2	1.33	8.31
85 234....	7.0	7506	7.85	WG−	G4	1.05	8.90
85 222....	7.0	7281	7.36	GW+	F5	0.63	7.99
84 311....	7.5	7331	7.42	WG−	F9	0.82	8.24
83 397....	6.5	7459	6.07	WG	G3	1.00	7.07

TABLE VII—*Continued*

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
1...	8.1	9.28	C	3	0.01	12...	9.9	11.11	R	7	0.07
2...	8.5	8.95	R	4	0.04	13...	10.1	11.51	R	5	0.08
3...	8.7	9.20	R	5	0.04	14...	10.4	11.69	R	4	0.06
4...	8.7	9.01	C	3	0.02	15...	10.6	11.93	R	2	....
5...	9.0	10.85	R	1	....	16...	10.7	12.3	R	1	....
6...	9.0	9.48	C	3	0.02	19...	11.2	12.9	R	1	....
7...	9.0	10.53	R	7	0.04	21...	11.3	12.5	R	1	....
8...	9.4	10.17	R	3	0.08	22...	11.4	12.6	R	1	....
9...	9.7	10.67	R	3	0.10	23...	11.6	13.1	R	1	....
10...	9.9	11.36	R	7	0.04	24...	11.7	12.7	R	1	....
11...	9.9	10.88	R	7	0.05	25...	11.9	13.0	R	1	....

that they too are reduced to the mean of the four. Because of the position of this field it was impossible to make any more exposures on it in the time remaining for the work.

In this field it was impossible to find a sufficient number of white stars to be used as standards. Consequently, in addition to the one star of spectral type A1, six other stars from the *P.D.* list whose spectra were well determined from objective-prism plates were used. The corrections for the spectral type were applied according to Parkhurst's curve. It is of interest to note that the photographic magnitudes of the Hagen stars 1, 4, and 6 based upon these standards differ by only 0.02, 0.03, and 0.01, respectively, from the magnitudes obtained for these stars if based directly upon the magnitudes of the standards as given in Parkhurst's *Zone Catalogue*.

TABLE VIII

RU HERCULIS FIELD

$\alpha = 16^h 6^m 3^s$ $\delta = +25° 19'.9$ (1900) Hagen Series VI

STANDARD STARS

B.D.		*P.D.*			Spectrum	Cor. for Spectrum	Photographic Magnitude
No.	Mag.	No.	Mag.	Color			
23°2916...	7.0	8599	6.86	WG	G5	1.08	7.94
27 2597...	7.3	8554	7.86	GW−	F0	0.40	8.26

TABLE VIII—*Continued*

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
1...	...	8.21	C	4	0.03	19...	10.3	11.95	R	4	0.08
3...	8.5	10.02	R	6	0.04	20...	10.3	12.33	R	4	0.02
4...	8.7	9.46	R	6	0.09	21...	10.4	12.82	R	2	....
5...	8.8	9.49	R	5	0.06	22...	10.4	12.49	R	3	0.11
6...	8.9	9.98	R	5	0.06	23...	10.5	12.39	R	2	....
7...	9.0	10.6	R	1	....	24...	10.5	12.16	R	4	0.02
9...	9.4	10.82	R	6	0.04	25...	10.5	12.30	R	3	0.02
10...	9.5	11.5	R	1	....	26...	10.6	12.3	R	1	....
11...	9.5	11.5	R	1	....	27...	10.8	13.3	R	1	....
12...	9.6	11.03	R	6	0.04	29...	11.0	12.84	R	2	....
13...	9.8	12.2	R	1	....	30...	11.0	12.76	R	2	....
14...	9.8	11.54	R	6	0.08	31...	11.2	13.02	R	3	0.06
15...	10.0	11.9	R	1	....	32...	11.3	13.41	R	2	....
16...	10.0	11.64	R	6	0.07	33...	11.5	13.36	R	2	....
18...	10.2	12.21	R	4	0.07	34...	11.6	13.3	R	1	....
						35...	11.9	13.4	R	1	....

VISUAL MAGNITUDES

Hagen No.	Hagen Mag.	Magnitude	Color-Index	Spectrum Inferred	Instr.	No. of Plates	p.e.
1.....	...	7.50	0.71	F8	C	5	0.02

This field is exceptional in that no white, or even approximately white, stars, bright enough to be in the *P.D.*, appeared on the camera plate. The stars used as standards, as the spectrum column shows, are advanced beyond the stage of those usually used. The

TABLE IX

W HERCULIS FIELD

$\alpha = 16^h 31^m 41^s$ $\delta = +37^\circ 32'.4$ (1900) Hagen Series III

STANDARD STARS

B.D.		*P.D.*			Spectrum	Cor. for Spectrum	Photographic Magnitude
No.	Mag.	No.	Mag.	Color			
37°2750...	5.7	8670	5.82	W+	A3	0.12	5.94
38 2811...	7.2	8785	7.72	GW	A2	0.08	7.80
37 2802...	7.5	8851	7.47	W+	Ao	0.00	7.47

TABLE IX—*Continued*

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
1...	7.9	9.04	C	3	0.01	14...	9.5	9.95	R	3	0.02
2...	8.2	8.75	C	3	0.02	15...	9.5	11.31	R	3	0.01
3...	8.3	9.22	R	3	0.06	16...	9.8	11.48	R	3	0.07
4...	8.4	8.95	C	3	0.02	17...	9.9	11.02	R	3	0.04
5...	8.4	8.44	C	3	0.02	18...	10.0	11.08	R	3	0.07
6...	8.6	9.87	R	3	0.08	19...	10.3	11.79	R	4	0.01
7...	8.8	9.87	R	3	0.08	20...	10.4	11.40	R	3	0.06
8...	8.9	10.28	R	3	0.07	21...	10.5	11.58	R	3	0.07
9...	9.0	10.78	R	3	0.04	22...	10.7	12.36	R	2	....
10...	9.2	10.98	R	3	0.04	24...	11.0	12.98	R	2	....
11...	9.2	10.28	R	3	0.04	25...	11.2	12.58	R	2	....
12...	9.3	10.84	R	3	0.04	27...	11.4	12.96	R	2	....
13...	9.4	10.64	R	3	0.06	28...	11.4	12.84	R	3	0.08
						29...	11.6	12.89	R	3	0.05

VISUAL MAGNITUDES

Hagen No.	Hagen Mag.	Magnitude	Color-Index	Spectrum Inferred	Instr.	No. of Plates	p.e.
1.....	7.9	7.71	+1.33	Ko	C	3	0.02
2.....	8.2	8.33	+0.42	Fo	C	3	0.03
4.....	8.4	8.62	+0.33	A8	C	3	0.01
5.....	8.4	8.62	−0.18	B7	C	3	0.05

estimates of these spectral types were verified by Parkhurst and the corrections applied from the most recent curve so that the magnitudes in this field are, nevertheless, on the same basis as those of the other fields in this list.

TABLE X

X CYGNI FIELD

$\alpha = 19^h46^m44^s$ $\delta = +32°39'.7$ (1900) Hagen Series III

STANDARD STARS

B.D.		*P.D.*			Spectrum	Cor. for Spectrum	Photographic Magnitude
No.	Mag.	No.	Mag.	Color			
33°3572...	6.7	10960	6.38	W	Ao	0.00	6.38
32 3662...	7.0	11171	7.52	W+	Ao	0.00	7.52
31 3779...	6.5	11003	7.11	W+	Ao	0.00	7.11
35 3786...	7.0	10978	6.88	GW	Ao	0.00	6.88
34 3778...	6.8	11084	7.38	W	Ao	0.00	7.38

TABLE X—*Continued*

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
3...	...	6.64	C	3	0.03	58...	10.1	10.71	R	3	0.04
6...	8.3	8.67	R	3	0.02	59...	10.1	11.46	R	3	0.09
7...	8.5	8.15	C	3	0.04	60...	10.1	10.83	R	3	0.06
8...	8.6	8.6	R	1	....	62...	10.2	11.32	R	3	0.08
9...	8.6	8.51	C	3	0.05	63...	10.2	10.90	R	3	0.01
11...	8.7	9.74	R	2	....	64...	10.2	10.80	R	3	0.01
12...	8.8	10.53	R	3	0.04	65...	10.3	11.23	R	3	0.05
13...	8.8	8.95	C	3	0.04	66...	10.3	11.02	R	3	0.02
14...	8.9	8.99	R	3	0.08	67...	10.4	11.12	R	3	0.06
15...	9.0	10.11	R	2	....	68...	10.4	11.28	R	3	0.05
16...	9.0	9.66	R	5	0.08	69...	10.4	11.81	R	3	0.06
17...	9.0	9.77	R	3	0.05	70...	10.4	11.92	R	3	0.11
18...	9.0	9.33	R	3	0.05	71...	10.5	11.59	R	3	0.04
19...	9.0	10.83	R	3	0.04	72...	10.5	11.31	R	3	0.03
20...	9.1	8.95	R	3	0.03	73...	10.5	11.22	R	3	0.04
21...	9.1	10.87	R	3	0.10	74...	10.5	11.47	R	3	0.05
23...	9.1	9.72	R	3	0.06	75...	10.6	11.58	R	3	0.08
24...	9.2	9.09	R	2	....	76...	10.6	11.88	R	3	0.04
25...	9.2	9.37	R	3	0.07	79...	10.8	11.68	R	3	0.08
26...	9.2	9.37	R	3	0.04	81...	10.8	12.01	R	3	0.07
27...	9.3	10.73	R	3	0.05	82...	10.9	12.04	R	3	0.08
28...	9.3	9.46	R	3	0.04	83...	10.9	12.06	R	3	0.08
30...	9.3	9.77	R	3	0.09	84...	11.0	11.63	R	3	0.06
31...	9.4	10.05	R	3	0.04	85...	11.0	11.51	R	3	0.04
34...	9.5	9.77	R	3	0.04	86...	11.1	11.72	R	3	0.03
35...	9.5	9.91	R	3	0.11	87...	11.1	11.73	R	3	0.04
36...	9.5	9.79	R	3	0.06	88...	11.2	11.95	R	3	0.06
37...	9.5	10.00	R	2	....	89...	11.2	12.68	R	3	0.07
39...	9.6	10.39	R	2	....	90...	11.2	11.74	R	3	0.07
41...	9.7	10.41	R	3	0.02	91...	11.4	11.86	R	3	0.05
43...	9.7	10.25	R	3	0.06	95...	11.6	11.90	R	3	0.06
44...	9.7	10.67	R	2	....	96...	11.6	12.30	R	3	0.08
46...	9.8	10.44	R	3	0.08	97...	11.7	12.30	R	3	0.09
47...	9.8	10.98	R	3	0.06	98...	11.7	12.47	R	3	0.08
49...	9.9	10.76	R	3	0.04	100...	11.7	12.60	R	3	0.07
50...	9.9	10.34	R	3	0.04	101...	11.7	12.44	R	3	0.07
51...	9.9	11.16	R	3	0.06	104...	11.9	12.12	R	3	0.08
54...	10.0	11.06	R	3	0.05	106...	12.0	12.38	R	3	0.02
55...	10.1	10.62	R	3	0.07	107...	12.0	12.91	R	3	0.09
56...	10.1	10.68	R	3	0.05	108...	12.1	12.04	R	3	0.03
57...	10.1	11.19	R	3	0.07	109...	12.2	12.50	R	3	0.04
						110...	12.4	11.82	R	3	0.02

PLATE V

SPECIMEN PLATE OF THE *S Cygni* REGION, TAKEN AT THE FOCUS OF THE 24-INCH REFLECTOR OF THE YERKES OBSERVATORY

TABLE X—*Continued*

VISUAL MAGNITUDES

Hagen No.	Hagen Mag.	Magnitude	Color-Index	Spectrum Inferred	Instr.	No. of Plates	p.e.
3.....	...	6.54	+0.10	A3	C	5	0.05
7.....	8.5	8.33	−0.18	B7	C	3	0.04
9.....	8.6	8.46	+0.05	A2	C	2	....
13.....	8.8	8.71	+0.24	A6	C	1	....

TABLE XI

S CYGNI FIELD

$\alpha = 20^h 3^m 24^s$ $\delta = +57°41'.9$ (1900) Hagen Series VI

STANDARD STARS

B.D.		*P.D.*			Spectrum	Cor. for Spectrum	Photographic Magnitude
No.	Mag.	No.	Mag.	Color			
59°2137...	5.8	11107	6.26	GW−	A0	0.00	6.26
57 2084...	5.4	11100	5.28	W+	A0	0.00	5.28
56 2331...	6.3	11144	6.38	GW−	A0	0.00	6.38

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
1...	7.2	7.04	C	3	0.01	35...	9.9	11.42	R	3	0.03
2...	7.6	8.37	R	3	0.04	36...	10.0	10.61	R	3	0.12
4...	7.7	7.18	C	3	0.01	38...	10.0	11.10	R	3	0.03
6...	8.4	9.20	R	3	0.03	39...	10.1	10.72	R	3	0.12
7...	8.5	10.27	R	3	0.01	40...	10.1	11.89	R	3	0.06
8...	8.6	9.24	R	2	....	41...	10.2	11.45	R	3	0.03
9...	8.6	9.80	R	3	0.05	42...	10.2	11.66	R	3	0.02
11...	8.7	9.74	R	3	0.06	43...	10.3	11.19	R	3	0.08
12...	8.8	9.35	R	3	0.04	44...	10.3	11.78	R	3	0.08
15...	8.9	10.23	R	3	0.12	45...	10.3	11.35	R	3	0.06
16...	9.0	9.47	R	3	0.04	46...	10.3	11.44	R	3	0.04
17...	9.0	10.28	R	3	0.03	47...	10.4	10.86	R	6	0.03
18...	9.1	9.87	R	6	0.02	48...	10.4	11.21	R	3	0.07
20...	9.2	9.62	R	3	0.07	49...	10.4	12.11	R	3	0.01
22...	9.3	10.53	R	3	0.04	50...	10.5	11.43	R	3	0.05
23...	9.3	9.59	R	3	0.04	51...	10.5	11.20	R	3	0.03
24...	9.4	10.53	R	3	0.04	52...	10.5	11.76	R	3	0.03
25...	9.4	9.76	R	6	0.03	53...	10.6	11.21	R	3	0.05
26...	9.4	10.41	R	3	0.10	54...	10.6	12.04	R	3	0.07
27...	9.4	9.67	R	3	0.06	55...	10.6	11.77	R	3	0.03
28...	9.4	10.95	R	3	0.03	56...	10.6	11.29	R	3	0.03
29...	9.5	11.13	R	3	0.07	59...	10.7	11.35	R	3	0.03
30...	9.6	10.28	R	3	0.06	64...	10.9	11.95	R	3	0.05
33...	9.7	9.75	R	3	0.06	68...	11.1	11.75	R	3	0.04
34...	9.7	10.72	R	3	0.09	75...	11.4	12.19	R	3	0.08

TABLE XI—*Continued*

VISUAL MAGNITUDES

Hagen No.	Hagen Mag.	Magnitude	Color-Index	Spectrum Inferred	Instr.	No. of Plates	p.e.
1.....	7.2	7.02	+0.02	A1	C	3	0.03
2.....	7.6	7.63	+0.74	F7	C	3	0.13
4.....	7.7	7.38	−0.20	B6	C	3	0.05

TABLE XII

V DELPHINI FIELD

$\alpha = 20^h43^m14^s$ $\delta = +18°58'.0$ (1900) Hagen Series VI

STANDARD STARS

B.D.		*P.D.*			Spectrum	Cor. for Spectrum	Photographic Magnitude
No.	Mag.	No.	Mag.	Color			
17°4378...	6.8	11782	7.03	GW	A0	0.00	7.03
17 4431...	6.5	11903	6.98	GW	A2	0.08	7.06
15 4220...	6.0	11752	6.30	W	A0	0.00	6.30

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
2...	8.3	8.30	R	3	0.04	23...	9.9	11.04	R	3	0.06
3...	8.5	8.43	R	3	0.10	24...	10.0	10.73	R	3	0.07
4...	8.7	8.90	C	4	0.02	25...	10.1	11.21	R	3	0.06
5...	8.8	8.80	C	5	0.02	26...	10.2	11.53	R	3	0.04
6...	8.9	9.79	R	3	0.04	28...	10.3	11.53	R	3	0.05
7...	8.9	8.99	R	3	0.08	29...	10.4	11.44	R	3	0.04
8...	9.0	9.94	R	3	0.02	30...	10.5	11.30	R	3	0.03
9...	9.1	9.74	R	3	0.05	31...	10.5	11.90	R	3	0.08
10...	9.1	9.96	R	3	0.06	32...	10.5	12.01	R	3	0.11
11...	9.1	9.27	R	3	0.00	33...	10.6	11.74	R	3	0.02
12...	9.2	9.61	R	3	0.01	34...	10.6	11.24	R	3	0.11
13...	9.3	9.64	R	3	0.09	35...	10.7	11.77	R	3	0.08
14...	9.4	10.73	R	3	0.05	36...	10.7	11.34	R	3	0.03
15...	9.5	10.62	R	3	0.05	37...	10.7	11.60	R	3	0.07
16...	9.6	9.95	R	3	0.06	38...	10.7	11.19	R	3	0.04
17...	9.7	9.10	R	3	0.04	39...	10.9	11.54	R	3	0.02
18...	9.7	10.57	R	3	0.07	40...	10.9	11.65	R	3	0.03
19...	9.7	11.06	R	3	0.03	41...	10.9	11.58	R	3	0.05
21...	9.8	10.13	R	3	0.06	43...	11.0	11.56	R	2	....
22...	9.9	10.30	R	3	0.06	46...	11.1	12.07	R	3	0.03

TABLE XIII

R LACERTAE FIELD

$\alpha = 22^h38^m50^s$ $\delta = +41°50'.9$ (1900) Hagen Series III

STANDARD STARS

B.D.		*P.D.*			Spectrum	Cor. for Spectrum	Photographic Magnitude
No.	Mag.	No.	Mag.	Color			
43°4298...	7.1	13348	7.18	W+	A0	0.00	7.18
40 4926...	7.0	13410	7.24	W+	A0	0.00	7.24
43 4258...	7.0	13264	7.40	GW	A0	0.00	7.40
43 4331...	6.0	13435	5.97	GW	A3	0.12	6.09
41 4619...	7.0	13387	7.45	GW−	A3	0.12	7.57
40 4866...	7.1	13258	7.76	W+	A0	0.00	7.76

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
2...	8.0	8.55	C	4	0.05	31...	10.2	12.93	R	3	0.04
5...	8.4	8.99	C	4	0.10	35...	10.5	11.52	R	3	0.03
6...	8.4	9.73	R	4	0.06	38...	10.7	11.11	R	3	0.02
7...	8.6	8.77	C	4	0.06	39...	10.8	11.04	R	3	0.04
8...	8.8	9.86	R	4	0.06	42...	11.0	11.41	R	3	0.03
9...	8.8	9.11	R	4	0.03	43...	11.1	11.22	R	3	0.08
11...	9.0	9.11	C	4	0.08	44...	11.1	12.08	R	3	0.02
12...	9.1	10.11	R	4	0.04	46...	11.1	12.00	R	3	0.05
14...	9.2	9.68	R	4	0.04	47...	11.2	11.96	R	3	0.01
15...	9.3	9.64	R	4	0.02	49...	11.2	11.76	R	3	0.02
16...	9.4	10.58	R	4	0.06	50...	11.3	11.88	R	2	....
18...	9.5	10.03	R	3	0.01	52...	11.3	12.24	R	3	0.04
19...	9.5	9.76	R	4	0.05	57...	11.5	12.13	R	3	0.04
22...	9.6	9.87	R	2	....	58...	11.5	12.10	R	2	....
23...	9.6	10.50	R	3	0.02	61...	11.6	12.57	R	3	0.02
26...	9.9	10.32	R	4	0.03	63...	11.7	11.90	R	3	0.05
28...	10.0	10.72	R	3	0.06	65...	11.8	12.08	R	3	0.02
29...	10.1	11.80	R	3	0.06	67...	12.0	13.00	R	2	....
30...	10.2	10.26	R	3	0.12	68...	12.0	12.57	R	2	....
						70...	12.2	12.64	R	3	0.05

VISUAL MAGNITUDES

Hagen No.	Hagen Mag.	Magnitude	Color-Index	Spectrum Inferred	Instr.	No. of Plates	p.e.
2.....	8.0	7.70	+0.85	F9	C	5	0.03
5.....	8.4	8.31	+0.68	F5	C	4	0.05
7.....	8.6	8.52	+0.25	A6	C	1	....

TABLE XIV

R PEGASI FIELD

$\alpha = 23^h 1^m 38^s$ $\delta = +10°0'.2$ (1900) Hagen Series II

STANDARD STARS

B.D.		*P.D.*			SPECTRUM	COR. FOR SPECTRUM	PHOTOGRAPHIC MAGNITUDE
No.	Mag.	No.	Mag.	Color			
19°5170...	5.5	13618	5.41	GW−	A0	0.00	5.41
11 4904...	6.9	13457	6.84	GW	A2	0.08	6.92
8 4961...	4.7	13449	5.08	GW−	A0	0.00	5.08
6 5092...	7.3	13481	6.60	GW	A0	0.00	6.60
7 4991...	6.0	13643	5.39	GW	A2	0.08	5.47
6 5124...	7.5	13619	7.63	WG−	A2	0.08	7.71

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
1...	8.1	7.84	C	3	0.06	18...	10.5	11.64	R	3	0.06
2...	8.1	7.52	C	3	0.06	19...	10.7	12.04	R	2	.....
3...	8.4	9.36	C	1	....	20...	10.8	12.00	R	2	.....
4...	8.6	8.98	C	2	....	21...	10.9	11.77	R	2	.....
9...	9.4	10.35	R	3	0.10	22...	11.0	11.74	R	2	.....
10...	9.4	10.72	R	3	0.01	23...	11.1	12.50	R	2	.....
12...	9.8	10.59	R	3	0.06	24...	11.3	12.49	R	2	.....
13...	9.9	10.79	R	3	0.02	25...	11.3	12.12	R	2	.....
15...	10.2	11.41	R	3	0.05	26...	11.6	12.00	R	2	.....
16...	10.3	11.39	R	3	0.01	29...	11.9	12.36	R	2	.....
17...	10.4	11.48	R	3	0.05	30...	12.0	12.26	R	2	.....
						32...	12.3	12.19	R	2	.....

VISUAL MAGNITUDES

Hagen No.	Hagen Mag.	Magnitude	Color-Index	Spectrum Inferred	Instr.	No. of Plates	p.e.
1.....	8.1	7.60	+0.24	A6	C	4	0.04
2.....	8.1	7.23	+0.29	A7	C	4	0.02

TABLE XV

Y CASSIOPEIAE FIELD

$\alpha = 23^h58^m14^s$ $\delta = +55°7'.5$ (1900) Hagen Series VI

STANDARD STARS

B.D. No.	*B.D.* Mag.	*P.D.* No.	*P.D.* Mag.	*P.D.* Color	Spectrum	Cor. for Spectrum	Photographic Magnitude
56° 11...	6.7	52	6.86	W	A2	0.08	6.94
55 15...	7.4	62	7.52	W+	Ao	0.00	7.52
56 31...	7.2	127	7.82	GW−	Ao	0.00	7.82
56 3127...	7.4	14128	7.35	W+	Ao	0.00	7.35

MEASURED STARS

PHOTOGRAPHIC MAGNITUDES

Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.	Hagen No.	Hagen Mag.	Photog. Mag.	Instr.	No. of Plates	p.e.
2...	7.5	7.97	C.R	8	0.05	22...	9.7	10.06	R	4	0.10
3...	7.9	7.21	C	3	0.02	23...	9.8	9.97	R	4	0.06
4...	8.3	8.60	C	3	0.01	24...	9.9	10.36	R	3	0.10
6...	8.6	9.98	R	4	0.06	25...	10.0	10.40	R	4	0.05
7...	8.8	9.60	R	4	0.06	26...	10.1	10.19	R	4	0.08
9...	8.9	8.94	R	4	0.05	27...	10.2	11.14	R	3	0.08
10...	9.0	9.16	R	4	0.05	28...	10.3	11.1	R	1	....
11...	9.0	8.91	R	4	0.04	29...	10.3	10.26	R	4	0.06
12...	9.0	10.49	R	3	0.05	31...	10.3	10.54	R	4	0.10
13...	9.1	9.01	R	4	0.08	33...	10.4	11.00	R	4	0.06
14...	9.2	9.37	R	4	0.08	34...	10.4	10.91	R	4	0.09
15...	9.2	9.57	R	4	0.03	36...	10.5	10.52	R	4	0.04
16...	9.4	9.70	R	4	0.09	37...	10.5	10.58	R	4	0.10
17...	9.4	10.64	R	4	0.06	38...	10.5	10.68	R	4	0.07
18...	9.5	10.45	R	4	0.07	40...	10.6	10.5	R	1	....
20...	9.6	10.61	R	4	0.08	52...	11.0	11.68	R	4	0.09
21...	9.6	9.90	R	4	0.04						

VISUAL MAGNITUDES

Hagen No.	Hagen Mag.	Magnitude	Color-Index	Spectrum Inferred	Instr.	No. of Plates	p.e.
2.....	7.5	6.88	+1.09	G5	C	3	0.03
3.....	7.9	7.35	−0.14	B8	C	3	0.02
4.....	8.3	7.94	+0.66	F5	C	3	0.03

The thanks of the author are due and are hereby most sincerely expressed to the various persons who have contributed to the progress of this work; to Professor Frost, who from the beginning showed an interest in the plan and very kindly permitted the use of the instruments necessary to the success of the undertaking; to all the members of the Observatory staff, who by their generous hospitality and help have made the time spent at the Observatory most pleasurable; to Mr. R. H. Motherwell, who made several exposures with the camera; to Mr. H. L. Alden, who assisted in a part of the reductions; and especially to Professor Parkhurst, who suggested the problem, and who, in the midst of the difficulties incident to the work, has been an inspiration not only because of the actual efforts he put forth to further the plan but also because of patience, the helpful suggestions, and the sympathetic and enthusiastic interest which he always manifested.

WILLIAMS BAY, WISCONSIN
YERKES OBSERVATORY
August 22, 1912

THE PHOTOGRAPHIC MAGNITUDE SCALE OF THE NORTH POLAR SEQUENCE[1]

By FREDERICK H. SEARES

During the past two years various investigations have been undertaken at the Mount Wilson Solar Observatory for the purpose of establishing methods of photographic photometry that may safely be employed with the 60-inch reflector. Those finally adopted involve the use of wire gauze screens and diaphragms of various apertures.

The method of diaphragms has in the past been considered questionable because of the uncertainty as to the effect of the change in the diffraction pattern. This has been investigated by two different methods and the evidence indicates that with the apertures used the diffraction effect can scarcely exceed two- or three-hundredths of a magnitude in a range of five magnitudes. The reduction constants of the diaphragms have accordingly been calculated directly from the measured areas of the apertures.

The wire gauze screen is free from difficulties arising from diffraction; but the determination of its absorption constant requires a laboratory investigation, and, without special precautions, is subject to systematic error. To obtain definite assurance of the reliability of the adopted constants, both the point and the surface absorption were measured. The point absorption was determined by means of a specially devised photometer. The surface absorption was found with the aid of an instrument of the Lummer-Brodhun type and by calculating from the measured dimensions of the mesh of the screen. The surface absorption was further controlled by the use of widely different bench lengths and by the measurement of double as well as single thicknesses of the screen. The mean result of the pieces investigated was: point absorption, 3.038 mags.; surface absorption 1.503 mags. The former differs from twice the latter by only 0.03 mag.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 70.

An objection affecting the use of both diaphragms and screens arises from the fact that the exposures of reduced intensity cannot be made simultaneously with those of full aperture. This doubles the time required for the observations and introduces errors having their origin in varying atmospheric conditions. The increased time of exposure is objectionable, but in itself does not affect the precision of the results. Changes in atmospheric transparency and steadiness are serious, though with the short exposures possible with the 60-inch reflector the chance of error is lessened, and the disturbance may be rendered accidental by increasing the number of plates. For control, the various exposures with full aperture, diaphragms, and screens are always symmetrically arranged. It is therefore possible practically to reduce the atmospheric difficulty to one of time.

The most serious difficulty encountered has been the satisfactory determination of the error depending on the distances of the stars from the axis of the instrument. It was expected that, on account of the large ratio of aperture to focal length, the corrections would be large, but that they would fluctuate from plate to plate in a most erratic manner was not foreseen. The influences of errors of focus, of the time, concentration, and temperature of development were successively studied without the discovery of any adequate explanation. With the appearance of irregularities in the error for a single plate when different directions from the axis were considered, it was suspected that temperature deformations of the mirror might be involved. A photographic record of the figure of the mirror by the knife-edge method was then made simultaneously with determinations of the distance correction. Under normal conditions it was found that the correction behaved normally; but with the greatly disturbed figure purposely produced to increase the decisiveness of the test, it became quite unmanageable.

Formerly it was not possible to cover the telescope with the canopy when the Cassegrain spectrograph was in position, and the arrangement of the program was such that the photometric observations followed those with the spectrograph. Part of the photometric plates were therefore made with the mirror in an abnormal condition, and it was these which first revealed the irregularities.

Although the distance correction is appreciably affected by even small changes of figure, experience indicates that with a careful temperature control reliable photometric results may be obtained through the use of an average value of the correction.

As with the refractor, the distance error is such as to increase the brightness of the brighter stars and decrease that of the fainter. Its variation with brightness is roughly proportional to the magnitude, although with the fainter objects the change is more rapid than with the brighter. The dependence upon distance from the axis is sensibly linear, which is in agreement with the theoretical aberration of the reflector. The correction is so little affected by errors of focus that with ordinary care this source of error may be disregarded; and it seems to be in no wise dependent on the conditions of development.

The first serious attempt at the determination of magnitudes has been for the stars of the North Polar Sequence, and as the question of greatest immediate interest in photographic photometry is concerned with the determination of an absolute scale, the results here described are considered from this standpoint. The work has naturally divided itself into two parts, the first of which relates to the stars fainter than magnitude 10.5. The second includes the brighter stars.

Preliminary results for the fainter stars were presented at the fourteenth meeting of the Astronomical and Astrophysical Society of America. They depend upon 15 plates made with the wire gauze screen and with diaphragms of 32 and 14 inches, and include 27 separate determinations of the scale. The exposures varied from 1 to 11 minutes and gave measurable images from magnitude 8.8 to 17.6, though the limits of reliable determination are 10.5 and 15.5.

The scale found for this interval is in close agreement with that derived at the Harvard Observatory, and on this account only a summary of the results is given here. The difference in the two scales, calculated with the Harvard magnitudes of *H.C.*, **170**, is shown in Table I.

The zero point of the preliminary Mount Wilson scale has been chosen so that the algebraic sum of the deviations between magni-

tudes 10.5 and 15.5 is zero. The last two differences are of small weight because the fainter stars of the group are shown only on the plates of longer exposure. There is also a possibility of systematic error arising from the fact that the images of these objects are at the limit of visibility. Plates of considerably longer exposure will therefore be required for a reliable determination of the fainter part of the scale, and until these have been obtained, no opinion can be expressed as to the reality of the indicated divergence for the stars fainter than 15.5.

TABLE I

COMPARISON OF SCALES FOR FAINT STARS

Magnitude	Mt. W.−H.	No. Stars
10.5–11.5	−0m03	6
11.5–12.5	−0.02	4
12.5–13.5	+0.03	6
13.5–14.5	+0.03	5
14.5–15.5	−0.01	5
15.5–16.5	−0.13	4
16.5–17.6	−0.27	7

In the meantime, the second part of the investigation, involving an extension of the scale to the brighter stars of the sequence, has been undertaken. Although the material is still incomplete, the general character of the results is indicated with sufficient clearness to justify a preliminary statement of the methods and conclusions.

The starting-point for this part of the work was the portion of the scale already determined. Between 10.5 and 15.5 the magnitudes of the stars of the sequence are assumed to be accurately known, although the values actually used are included within the limits 10.5 and 13.7. For this region the Harvard values were adopted, as they are based upon much more extensive data than were used for the preliminary Mount Wilson magnitudes.

The method employed was to photograph the brighter stars with various combinations of diaphragms and wire gauze screens so adjusted as to reduce them to an apparent brightness falling within the interval of known magnitudes. The apparent magnitudes were then determined by comparison with the stars of known brightness

photographed on the same plate, with the same exposure as the brighter stars, but with the full aperture of the telescope. The subtraction of the reduction constant of the diaphragms and screens from the apparent magnitudes gave the real magnitudes of the bright stars. Owing to the fact that the brighter stars of the sequence are scattered over a considerable area, while the field of good definition for a reflector of large ratio is limited, it has been necessary to investigate each of the stars separately. But as the individual exposures did not exceed 2^m, the time required for the observational part of the work was not great. The exposures were always symmetrically arranged in the order: bright star, faint stars, bright star. For each setting on the bright star, from two to four exposures were made with different combinations of diaphragms and screens arranged to give a considerable difference in the apparent magnitudes of the reduced images, and thus distribute them along the magnitude-curve defined by the faint stars of known brightness. As an illustration, from one plate for star No. 14, we have

Aperture	Appt. Mag.	Red. Const.	Mag.
40 in.....	11.69	1.12 mag.	10.57
32 in.....	12.53	1.96	10.57
14 in.....	13.42	2.97	10.45
Gauze screen.....	13.39	3.06	10.33

For the faint stars, the setting was always such as to bring the densest part of the group to the center of the plate.

The measurement of the plates was carried out by Miss High, of the Computing Division, although I have personally scrutinized all of the observational material for the rejection of defective images. I am also indebted to Miss High for much assistance in connection with the reductions.

The diaphragms used were of 40, 32, 14, 9, and 6 inches, having reduction constants of approximately 1, 2, 3, 4, and 5 magnitudes. The screens available were all made from bronze wire of the same mesh, having a point absorption of about 3 magnitudes. Two screens of 60 inches diameter, one of single thickness (G), and one of double thickness (G_2), were used. In addition, two smaller

pieces were available for use with the diaphragms of 14, 9, and 6 inches. A large number of different combinations was thus possible, and those actually employed range in absorption from 1 to 11 magnitudes. The absorption constants of the smaller pieces were determined by the methods referred to above. The large screens have not yet been directly measured. The preliminary values of their constants were derived from measures of a large number of remnants left from the construction of the screens. That of the large single thickness screen is certainly close to the true value, as is shown by various comparisons of results with the 14-inch diaphragm, whose constant differs from that of the screen by only 0.09 mag. The preliminary value used for the 60-inch double thickness screen is less reliable, but can scarcely be in error by as much as 0.1 mag. In any event, small systematic differences are to be expected as a matter of course, but the considerable number of methods used for the determination of the brightness of each star tends to render these accidental in character.

There is, however, a circumstance of importance which is independent of the difficulties raised by uncertainties in the absorption constants, namely, that it is possible to test the homogeneity of two isolated regions of a scale, even though the absorption of the screen or diaphragm be only approximately known. The magnitudes of one of the regions may remain uncertain relatively to those of the other by a constant amount, but if there be any difference in the slope of the two portions, it should be possible to bring this difference to light. For example, suppose that we wish to test the homogeneity of a given scale for the intervals 4–9 mags. and 10–15 mags., for which we have available a screen absorbing approximately 6 mags. With the aid of the screen, the brighter stars can be given an apparent brightness of from 10–15 magnitudes, which in each instance can be accurately determined by a comparison with the objects in the second region having actual magnitudes between 10 and 15. The subtraction of the assumed reduction constant from the observed apparent brightness then gives for the brighter stars magnitudes which will be homogeneous with respect to those of the fainter stars, though they may all be in error by a constant amount, namely, the error of the absorption constant. A comparison of the

screen results with the given magnitudes of the brighter stars will then reveal any lack of homogeneity in the two regions. The only assumption involved is that the absorption of the screen is the same for all of the stars lying within the interval 4–9 magnitudes.

The necessary operations for such a test are essentially the same as those described above for the determination of the magnitudes of the brighter stars. From this it follows that the slope of the scale derived for the bright stars by the use of any one of the combinations of screens or diaphragms should be the same as that for the adopted magnitudes between 10.5 and 15.5, although the results from a given aperture will, as a whole, be too bright or too faint by the amount of the error of the adopted absorption. But by using several combinations and covering long intervals, the different determinations will overlap, the systematic errors will assume an accidental character, and material will be accumulated for a discussion of the systematic errors affecting the various methods employed.

A summary of the results thus far accumulated is given in Table II. The first column contains the star numbers from *H.C.*, **170**; the second, the mean Mount Wilson magnitudes; and the twelve columns following, the deviations in hundredths from the mean magnitudes found with the arrangements of diaphragms and screens indicated by the table heading. The numbers in the first line of the heading refer to the apertures of the diaphragms; *G* is a single thickness of wire gauze, without specification of the particular piece used. The values in the fifth column were found with the 60-inch single thickness screen; those in the tenth, bearing the heading G_2, with the 60-inch double thickness screen. The numbers in the second line of the heading are the approximate values of the reduction constants.

The addition of the reduction constants to the mean magnitudes in the second column gives the approximate apparent magnitudes of the bright stars read from the magnitude-curves defined by the faint stars of known brightness. These indicate the particular part of the region of known magnitudes upon which the results depend. The actual value derived for any star with any given arrangement may be found by adding the deviation in the body of the table to

TABLE II

PRELIMINARY MEAN MAGNITUDES AND DEVIATIONS FOR VARIOUS APERTURES

No.	Mt. W. Mag.	40 1.1	32 2.0	G 3.1	14 3.0	9 3.9	32+G 5.0	6 4.8	G2 6.1	14+G 6.0	9+G 7.0	6+G 7.9	Various	Mt. W. Minus Harvard	Mt. W. Minus Yerkes	Mt. W. Minus Gött.
1S	2.10	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	(18)	−61	....	....
1	3.97	.....	.....	.....	.....	.....	.....	.....	.....	+18 (1)	− 6 (3)	+ 4 (3)	− 7 (2)	−50	−28	−37
2	4.76	.....	.....	.....	.....	.....	.....	.....	− 1 (5)	+ 2 (9)	−10 (2)	.....	.....	−48	−30	−38
3	5.36	.....	.....	.....	.....	.....	.....	.....	− 3 (7)	+ 5 (7)	0 (4)	− 8 (2)	.....	−38	−26	−34
4	5.56	.....	.....	.....	.....	.....	.....	.....	+ 3 (5)	+ 6 (6)	− 2 (5)	−16 (3)	.....	−35	−35	−36
2S	5.97	.....	.....	.....	.....	+10 (1)	.....	− 6 (3)	+ 1 (7)	− 1 (6)	+ 4 (3)	.....	.....	−41	−39	−42
5	5.99	.....	.....	.....	.....	.....	−13 (2)	+10 (3)	0 (3)	− 3 (4)	+ 1 (2)	.....	.....	−40	−44	−45
3S	6.21	.....	.....	.....	.....	− 2 (1)	− 9 (1)	+ 3 (4)	+18 (3)	− 2 (4)	−11 (4)	.....	.....	−33	−33	−38
1F	6.19	.....	.....	.....	.....	.....	.....	− 2 (4)	+ 7 (6)	0 (9)	−11 (4)	.....	.....	−50	−51	−47
6	6.74	.....	.....	.....	.....	−10 (1)	.....	−13 (3)	+18 (2)	+ 6 (2)	.....	.....	.....	−24	−38	−40
7	7.06	.....	.....	.....	.....	− 4 (2)	+ 7 (1)	+ 5 (2)	+ 2 (2)	− 6 (2)	.....	.....	.....	−12	−21	−22
2F	7.52	.....	.....	−20 (1)	−14 (1)	−27 (1)	+15 (5)	+ 5 (3)	.....	− 7 (3)	.....	.....	.....	−21	−57	−27
8	7.78	.....	.....	.....	− 8 (3)	− 6 (4)	.....	+18 (3)	+ 8 (4)	−10 (4)	.....	.....	.....	−32	....	....
3F	8.51	.....	+13 (1)	−24 (1)	− 7 (2)	+ 6 (3)	.....	+ 3 (2)	.....	.....	.....	.....	.....	−11	....	....
9	8.50	.....	.....	−12 (2)	− 4 (4)	+ 3 (5)	.....	+ 6 (4)	.....	.....	.....	.....	.....	−20	....	....
10	8.65	.....	−22 (2)	−17 (1)	+ 2 (3)	+15 (2)	.....	+22 (1)	.....	.....	.....	.....	.....	−24	....	....
4F	8.87	.....	− 4 (4)	.....	−15 (2)	− 5 (2)	+13 (4)	.....	.....	.....	.....	.....	.....	−10	....	....
11	9.19	.....	− 7 (3)	+ 1 (3)	+ 3 (3)	+ 4 (3)	.....	.....	.....	.....	.....	.....	.....	−23	....	....
12	9.83	.....	+10 (4)	+ 3 (1)	0 (4)	−20 (2)	.....	.....	.....	.....	.....	.....	.....	+10	....	....
5F	9.86	.....	+ 7 (4)	−11 (1)	− 4 (4)	.....	.....	.....	.....	.....	.....	.....	.....	+ 8	....	....
13	10.17	−31 (1)	+ 2 (3)	−15 (1)	+19 (2)	.....	.....	.....	.....	.....	.....	.....	.....	0	....	....
6F	10.07	− 4 (3)	+ 7 (3)	− 4 (2)	0 (2)	.....	.....	.....	.....	.....	.....	.....	.....	−10	....	....
14	10.51	+ 5 (2)	+11 (3)	−13 (2)	− 6 (3)	.....	.....	.....	.....	.....	.....	.....	.....	− 1	....	....
7F	10.54	− 4 (2)	+ 4 (2)	+10 (1)	−11 (1)	.....	.....	.....	.....	.....	.....	.....	.....	− 3	....	....
Mean deviations		− 5 (8)	+2 (29)	−8 (16)	−3 (34)	−1 (27)	+9 (13)	+3 (32)	+4 (44)	+0 (57)	−5 (27)	−6 (8)				

The Mount Wilson magnitudes in the second column and the differences in the last three columns may be referred to the international zero point by adding +0m37 throughout.

the mean magnitude. The quantities in parentheses indicate the number of determinations, each being based on two photographic images, one made before, the other after, the exposure on the faint stars. Thus, the mean value for the last star is based upon observations with diaphragms of 40 and 32 inches, the single thickness 60-inch wire gauze screen, and a 14-inch diaphragm. The approximate apparent magnitudes were 11.6, 12.5, 13.6, and 13.5 magnitudes, respectively. The mean values of the real brightness found with each combination were 10.50, 10.58, 10.64, and 10.43, and the number of determinations upon which these results depend are, respectively, 2, 2, 1, and 1.

The detailed results for No. 15 (*Polaris*), which do not appear in the table, are:

	Aperture			
	$G+G_1$	$14+2G$	$9+2G$	$6+2G$
Approximate absorption ...	9.2	9.2	10.0	10.9
Deviation.....	+2(3)	−3(4)	+3(7)	−3(4)

The weighted means of the deviations in each column appear in the last line of the table. It is difficult to interpret these in terms of systematic differences between the results found with the different apertures, as the grouping is not homogeneous. For example, the first residual in the 32-column is affected by the systematic errors of the *G*, 14, and 9 arrangements, while the last is influenced by those of the 40, *G*, and 14. The matter is further complicated by the fact that the mean values are still affected to a considerable degree by accidental errors due to the comparatively small number of plates used. Unless the apertures compared happen to have been used with the same plates throughout, the results are likely to be influenced by the systematic errors of the plates. Thus, the indicated systematic difference between the results for *G* and 14 is −0.05 mag. But if we restrict the comparison to the 11 plates upon which both arrangements were used, the difference becomes −0.02 mag. Similar comparisons of the scale for these two apertures based upon other material indicate that the true difference

does not exceed a hundredth of a magnitude. The chief significance of the mean deviations, therefore, is the indication they afford that the constant systematic errors affecting the various methods are relatively small.

The last three columns of the table contain the differences between the Mount Wilson mean magnitudes and those of Harvard, Yerkes, and Göttingen. The Harvard values are from *H.C.*, **170**, and those of Yerkes from the "Actinometry" of Parkhurst.[1] The Göttingen magnitudes are from the list of polar stars in the "Aktinometrie" of Schwarzschild.[2] All these represent independent determinations of the scale and are based upon radically different methods of observation and reduction. As the last two cover only a short interval, the discussion will be confined mainly to the comparison with the Harvard results.

The last four stars lie upon that part of the scale adjacent to the region of known magnitudes—in fact, the last two are among the stars whose brightness is assumed to be known. The present results for these four objects depend upon a comparison with stars of known magnitude between 11.2 and 13.6. That their mean deviations are small is an indication that, when applied to the region of known magnitudes, the methods employed for the bright stars give results in agreement with the original scale determination. But when the same methods (involving mainly the gauze screen, and the 32- and 14-inch diaphragms) are applied to slightly brighter stars, say of magnitude 8 to 9, the results no longer agree with those of Harvard, but show a gradually increasing divergence. That this is not due to systematic error in the determinations with the 9- and 6-inch diaphragms is clear, for the deviations of the overlapping portions are positive and make the differences between Mount Wilson and Harvard smaller than they would have been had these combinations been omitted in forming the mean magnitudes. Proceeding upward to the brighter stars, the divergence becomes even more marked. The differences gradually increase, and reach a value of 0.6 mag. for *Polaris*. When plotted with the magnitudes as abscissae, it is seen that the divergence is nearly

[1] *Astrophysical Journal*, **36**, 169, 1912.

[2] *Astronomische Mitteilungen der K. Sternwarte zu Göttingen*, **14** B, 42, 1912.

linear in character, and that it may be represented approximately by the formula

$$\text{Mt. Wilson} - \text{Harvard} = -0^{m}070\ (11.0 - M).$$

The results are shown graphically by the first curve in Fig. 1.

Although the effect of small errors in the absorption constants cannot now be fully determined, it seems clear that they have not contributed to any marked degree in determining the character of the divergence. The homogeneity test referred to above is immediately applicable to this question. Unless the effective absorption for the diaphragms and screens varies with the brightness of the star, the results given by any single combination must be homogeneous with those adopted for the fainter stars. But the residuals in each of the columns represent directly the differences between these supposedly homogeneous magnitudes and the mean magnitudes in the second column. In no instance is there a marked case of progression from large to small, or from positive to negative, values, and it therefore appears that the mean magnitudes are sensibly homogeneous with those adopted for the stars between 10.5 and 15.5.

Applying now the homogeneity test to the Harvard results, we have only to add the deviations in any column to the differences Mount Wilson−Harvard. The results will be the differences between the Harvard scale for various groups of bright stars and the presumably homogeneous magnitudes here derived for the same stars by the combination considered. If the differences be formed as indicated for each of the apertures, and then plotted with the corresponding magnitudes as abscissae, there results a series of curves in which are reproduced the characteristics of the divergence defined by the differences Mount Wilson−Harvard. Table III gives mean values of the differences formed for groups of stars, and the corresponding curves are shown in the lower part of Fig. 1. The character of the divergence is also shown for each case by the quantities in the last two columns of Table III which contain the constants of the linear equation best representing the deviations for the different series of determinations. The former of these is the amount of the divergence for an interval of one magnitude, and

TABLE III

COMPARISON OF SCALES FOR VARIOUS APERTURES WITH THE HARVARD SCALE

APERTURE	STARS	MEANS		NO. DET	DEV. FOR 1 MAG.	M_*
		Mag.	Mt.W.−H.			
32	3*r*, 10, 4*r*	8.76	−21	7	−0^{m}080	10.0
	11, 12, 5*r*	9.67	+ 5	11		
	13, 6*r*	10.12	0	6		
	14, 7*r*	10.52	+ 6	5		
Means		9.69	− 2			
G	2*r*, 3*r*, 9, 10	8.34	−36	5	−0.092	11.4
	11, 12, 5*r*	9.45	−11	5		
	13, 6*r*	10.10	−14	3		
	14, 7*r*	10.52	− 7	3		
Means		9.42	−19			
14	2*r*, 8	7.69	−39	4	−0.169	10.0
	3*r*, 9, 10, 4*r*	8.61	−23	11		
	11, 12, 5*r*	9.67	0	11		
	13, 6*r*, 14, 7*r*	10.32	− 2	8		
Means		9.25	−13			
9	2*s*, 3*s*, 6	6.31	−33	3	−0.119	9.9
	7, 2*r*, 8	7.52	−33	7		
	3*r*, 9, 10, 4*r*	8.59	−12	12		
	11, 12	9.45	−15	5		
Means		8.22	−20			
6	2*s*, 5, 3*s*	6.07	−35	10	−0.106	9.3
	6, 7	6.87	−25	5		
	2*r*, 8	7.64	−15	6		
	3*r*, 9, 10	8.52	−11	7		
Means		7.16	−23			
G_2	2	4.76	−49	5	−0.104	9.3
	3, 4	5.44	−37	12		
	2*s*, 5, 3*s*	6.03	−34	13		
	6, 7, 8	7.34	−16	8		
Means		5.95	−33			
14+*G*	1, 2, 3	4.90	−39	17	−0.026	19.8
	4, 2*s*	5.76	−36	12		
	5, 3*s*, 6	6.23	−35	10		
	7, 2*r*, 8	7.53	−32	9		
Means		5.88	−36			
9+*G*	1	3.97	−56	3	−0.119	9.1
	2	4.76	−58	2		
	3, 4	5.46	−37	9		
	2*s*, 5, 3*s*	6.09	−41	9		
Means		5.45	−43		−0.097	10.0

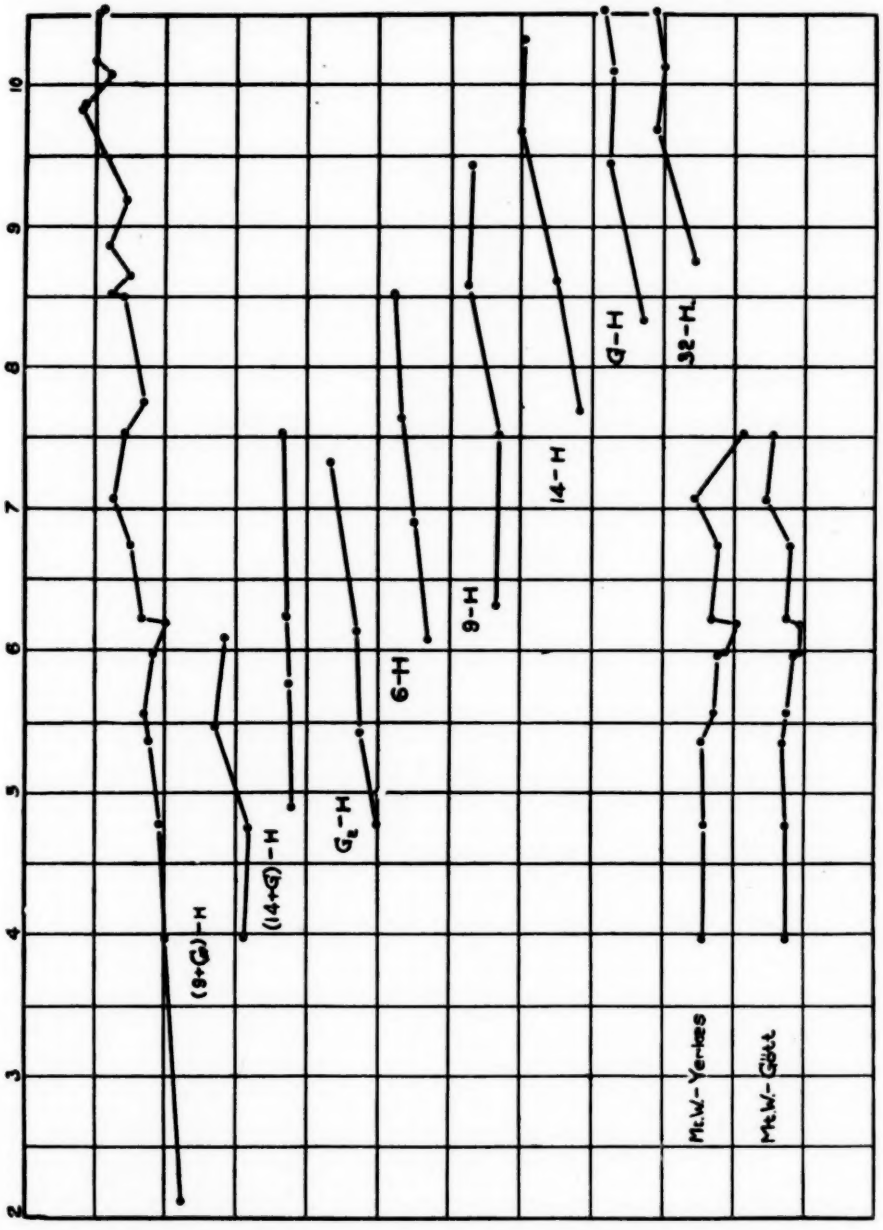

FIG. 1.—Comparison of Mount Wilson magnitudes with those of Harvard, Yerkes, and Göttingen. The first curve represents the mean values of Mt. W.—H. The curves below and to the right are the results derived from the combination of the aperture indicated with the full aperture of 60 inches. In general the ordinates are referred to the horizontal line immediately above as an axis. Vertical scale same as horizontal.

defines the inclination of the curves in Fig. 1, while the latter is the magnitude for which the scales derived by the different apertures agree with the Harvard scale. The irregularities are considerable, owing to the small number of determinations, and the inclination with respect to the axis varies from case to case. Nevertheless, the general features of the divergence are noticeable for each of the apertures 32, G, 14, 9, 6, G_2, 14+G, and 9+G. The intervals covered by the other determinations are too short to make them of service. The process is, in effect, an analysis of the mean curve of differences into its constituent parts, with the advantage that the inclinations of the constituent curves are not affected by the errors in the adopted reduction constants. These merely displace the separate curves in a vertical direction from their true position. The weighted mean divergence derived from the quantities in the sixth column of Table III is −0.097 mag.

The mean deviation for all the results derived with each aperture, and the corresponding mean magnitude are also appended to each of the groups in Table III. The mean deviations contain the effect of the errors in the adopted reduction constants, but it will be observed that they gradually increase with decreasing magnitude, and show a divergence of −0.069 mag. per magnitude. Were the errors of the reduction constants negligible, this quantity should equal the mean divergence of −0.097 mag. found above from the separate curves, and the difference is theoretically a measure of the effect of these errors. Actually, the influence is probably considerably less, for the values of the divergence found from the separate curves are subject to large uncertainties owing to the relatively short intervals covered. On the other hand, the uniformity with which the mean deviations for the different apertures increase with decreasing magnitude is an excellent test of the reliability of the reduction constants. Consider for example the results given by the single thickness gauze screen (G) and the 14-inch diaphragm, for which the mean magnitudes of the stars observed are practically the same. The difference in the resulting deviations is 0.06 mag. Again, for the double thickness screen (G_2), and the 14-inch diaphragm covered with a small piece of gauze (14+G), the difference in the mean deviations is only 0.03 mag. Incidentally, it may be

remarked that these two cases afford confirmation of the statement above concerning the freedom of the results obtained with diaphragms from any diffraction effect. With G and G_2 the central diffraction disk has the same diameter as that produced by the full aperture of 60 inches, while for 14 and 14+G it is four times as large. In one case the diaphragm results give the larger deviation, in the other, the smaller.

It appears, therefore, that whether we consider each aperture by itself, or the mean deviations as a group, we arrive at substantially the same result; and it is equally clear that errors in the reduction constants cannot have played any important part in producing the divergence between the Mount Wilson results and those of Harvard.

Finally, a word may be said with regard to the comparisons with Parkhurst and Schwarzschild, which are also shown graphically in Fig. 1. If the last, and apparently discordant, difference with Parkhurst be disregarded, there is little to be remarked in the way of divergence between Mount Wilson and Yerkes. Such as there is appears to be in a direction opposite to that given by the comparison with Harvard. With Göttingen, on the other hand, there is no positive evidence of a progressive difference. The constant difference of 0.37 mag. shown by both comparisons is due to the fact that Parkhurst and Schwarzschild have chosen their zero points to agree with the Harvard scale at the sixth magnitude, while for the preliminary Mount Wilson values the coincidence is between 10.5 and 15.5. The results may be reduced to the international standard by adding +0.37 mag. to the Mount Wilson magnitudes and to the ordinates of the various curves. The appearance of similar irregularities in the three series of differences, which is well shown in the figure, merely indicates that the Mount Wilson values for the individual stars are affected to a considerable extent by accidental errors. This is especially noticeable in the case of star 1*r* for which the Mount Wilson value, in spite of the considerable number of determinations, is probably abnormal. This object has been excluded from the means in Table III.

Since the Harvard magnitudes for the faint stars have been used for the derivation of the Mount Wilson results, it follows that

the application of the homogeneity test to the Harvard scale is in reality a comparison of Harvard magnitudes for bright stars with Harvard results for faint stars, the only intermediate step being the interposition of a screen or diaphragm for the purpose of giving the brighter objects an apparent magnitude comparable with the real brightness of the fainter stars. Only two conclusions seem possible: either the methods used for the derivation of the Mount Wilson magnitudes are subject to errors whose presence, upon a priori grounds, would not have been suspected, or else the Harvard scale is not wholly consistent.

The preceding paragraphs present the results from the standpoint of the principles that formed the basis of the observational program. The purpose was to obtain a scale for the brighter stars which should be homogeneous with an adopted series of magnitudes for the fainter objects, and to this end the scale for the faint stars was introduced as an essential factor into the investigation. The reduction constants also enter into the final results, but in such a way as to produce a partial neutralization of their errors. It is important to note, however, that the slope of the scale derived by any given combination of screens and diaphragms is quite independent of these errors.

It is possible, however, to analyze the data from an entirely different point of view, for they afford material for a series of scale determinations for the brighter stars which individually are independent of the magnitudes assumed for the fainter objects. The scale to be established is thus made to depend wholly upon the constants of the diaphragms and screens, the assumed magnitudes being used only to fix the zero point. That such an arrangement is possible follows from the fact that each bright star has been reduced to several different apparent magnitudes. Consequently, for a considerable number of bright stars there are values derived from approximately the same apparent magnitude, say the twelfth, and the collection of all such results gives at once a scale of the character described. The stars of the twelfth magnitude determine the zero point, but the scale depends entirely upon the diaphragms and screens used.

From this standpoint, therefore, the results are not merely an

extension of the scale assumed for the faint stars, but an essentially independent determination. The final mean result must obviously be the same as that already given, for it involves only a rearrangement of the data; but the analysis is of importance, for since each of the constituent scales thus derived is obtained by a comparison with faint stars of the same brightness, their individual slopes will be free from any influence depending on the apparent magnitude to which the bright stars were reduced. This is not true of the separate scales given above, and a comparison of the two series of results should bring to light any such influence.

This rearrangement of the material is shown in Table IV. As previously explained, the apparent magnitudes used for the derivation of the results in Table II may be found by adding to the mean magnitudes in the second column the approximate values of the reduction constants given in the table heading. These were formed and arranged in order of increasing values. Opposite each were written the corresponding star number, the mean Mount Wilson magnitude, the aperture used, and the deviation for that aperture given in the body of Table II. The resulting list was divided into seven parts, each of which, excepting the first and the last, includes a range in apparent magnitude of 0.4 mag. The quantities within each of the groups were then arranged in the order of decreasing brightness of the stars included. The results are those shown in Table IV. The significance, therefore, of any one of the groups of this table, say the second, is that the stars whose numbers and mean magnitudes appear in the second and third columns were reduced to apparent magnitudes lying between 10.8 and 11.2 by means of the diaphragms and screens listed in the fourth column. The comparison of the reduced images with faint stars of the same magnitude gave values for the actual brightness differing from the mean magnitudes by the amounts shown in the last column. As before, the unit in which the deviations are expressed is 0.01 mag., and the quantities in parentheses represent the number of determinations upon which each deviation depends.

If, therefore, to the mean magnitudes in the third column there be added the deviations in the last column, we shall have a scale whose slope is independent of the assumed scale for the faint stars

TABLE IV

DEVIATIONS ARRANGED ACCORDING TO APPARENT MAGNITUDE

Apparent Magnitude	Star	Mean Mt. W. Mag.	Aperture	Deviation
9.9–10.6.....	1	3.97	14+G	+18 (1)
	2s	5.97	9	+10 (1)
	3s	6.21	9	− 2 (1)
	6	6.74	9	−10 (1)
	2r	7.52	14	−14 (1)
	2r	7.52	G	−20 (1)
	3r	8.51	32	+13 (1)
	10	8.65	32	−22 (2)
			Mean.....	− 5 (9)
10.8–11.2.....	1	3.97	9+G	− 6 (3)
	2	4.78	14+G	+ 2 (9)
	2	4.78	G_2	− 1 (5)
	2s	5.97	6	− 6 (3)
	5	5.99	6	+10 (3)
	5	5.99	32+G	−13 (2)
	1r	6.19	6	− 2 (4)
	3s	6.21	6	+ 3 (4)
	3s	6.21	32+G	− 9 (1)
	7	7.06	9	− 4 ( )
	8	7.78	14	− 8 (3)
	4r	8.87	32	− 4 (4)
	11	9.19	32	− 7 (3)
	6r	10.07	40	− 4 (3)
			Mean.....	− 2 (49)
11.3–11.7.....	1s	2.10	G+G_2	+ 2 (3)
	1s	2.10	14+2G	− 3 (4)
	3	5.36	14+G	+ 5 (7)
	3	5.36	G_2	− 3 (7)
	4	5.56	14+G	+ 6 (6)
	4	5.56	G_2	+ 3 (5)
	6	6.74	6	−13 (3)
	2r	7.52	9	−27 (1)
	8	7.78	9	− 6 (4)
	9	8.50	14	− 4 (4)
	9	8.50	G	−12 (2)
	3r	8.51	14	− 7 (2)
	3r	8.51	G	−24 (1)
	10	8.65	14	+ 2 (3)
	13	10.17	40	−31 (1)
	14	10.51	40	+ 5 (2)
	7r	10.54	40	− 4 (2)
			Mean.....	− 2 (57)
11.8–12.2.....	1s	2.10	9+2G	+ 3 (7)
	1	3.97	6+G	+ 4 (3)
	2	4.76	9+G	−10 (2)
	2s	5.97	14+G	− 1 (6)
	2s	5.97	G_2	+ 1 (7)

TABLE IV—*Continued*

Apparent Magnitude	Star	Mean Mt. W. Mag.	Aperture	Deviation
11.8–12.2.....	5	5.99	G_2	0 (3)
	5	5.99	$14+G$	− 3 (4)
	1r	6.19	$14+G$	0 (9)
	3s	6.21	$14+G$	− 2 (4)
	7	7.06	6	+ 5 (2)
	7	7.06	$32+G$	+ 7 (1)
	10	8.65	G	−17 (1)
	4r	8.87	14	−15 (2)
	11	9.19	14	+ 3 (3)
	12	9.83	32	+10 (4)
	5r	9.86	32	+ 7 (4)
	6r	10.07	32	+ 7 (3)
	13	10.17	32	+ 2 (3)
			Mean.....	+ 1 (68)
12.3–12.7.....	3	5.38	$9+G$	0 (4)
	4	5.56	$9+G$	− 2 (5)
	1r	6.19	G_2	+ 7 (6)
	3s	6.21	G_2	+18 (3)
	6	6.74	$14+G$	+ 6 (2)
	2r	7.52	6	+ 5 (3)
	2r	7.52	$32+G$	+15 (5)
	8	7.78	6	+18 (3)
	9	8.50	9	+ 3 (5)
	3r	8.51	9	+ 6 (3)
	10	8.65	9	+15 (2)
	11	9.19	G	+ 1 (3)
	14	10.51	32	+11 (3)
	7r	10.54	32	+ 4 (2)
			Mean.....	+ 7 (49)
12.8–13.2.....	1s	2.10	$6+2G$	− 3 (4)
	1	3.97	$14+2G$	+ 7 (2)
	2s	5.97	$9+G$	+ 4 (3)
	5	5.99	$9+G$	+ 1 (2)
	1r	6.19	$9+G$	−11 (4)
	3s	6.21	$9+G$	−11 (4)
	6	6.74	G_2	+18 (2)
	7	7.06	$14+G$	− 6 (2)
	7	7.06	G_2	+ 2 (2)
	4r	8.87	9	− 5 (2)
	11	9.19	9	+ 4 (3)
	12	9.83	14	0 (4)
	12	9.83	G	+ 3 (1)
	5r	9.86	14	− 4 (4)
	5r	9.86	G	−11 (1)
	6r	10.07	14	0 (2)
	6r	10.07	G	− 4 (2)
	13	10.17	14	+19 (2)
			Mean.....	− 1 (46)

TABLE IV—*Continued*

Apparent Magnitude	Star	Mean Mt. W. Mag.	Aperture	Deviation
13.3–13.9.....	3	5.36	6+*G*	− 8 (2)
	4	5.56	6+*G*	−16 (3)
	2r	7.52	14+*G*	− 7 (3)
	8	7.78	14+*G*	−10 (4)
	8	7.78	G_2	+ 8 (4)
	9	8.50	6	+ 6 (4)
	3r	8.51	6	+ 3 (2)
	10	8.65	6	+22 (1)
	4r	8.87	32+*G*	+13 (4)
	12	9.83	9	−20 (2)
	13	10.17	*G*	−15 (1)
	14	10.51	14	− 6 (3)
	14	10.51	*G*	−13 (2)
	7r	10.54	14	−11 (1)
	7r	10.54	*G*	+10 (1)
			Mean.....	− 2 (37)

and depends only upon the constants of the screens and diaphragms used. The magnitudes of the faint stars have been used only to fix the zero point.

An examination of the deviations in the last column of Table IV shows at a glance that each of the seven scales is practically identical with the mean scale. There are irregularities, noticeable in the first group, caused by the small number of observations, and there are obviously constant systematic differences, but in no instance is there a progression in the deviations which would indicate an appreciable divergence of the separate scales from the mean. In other words, we arrive at the same result whether we use a single diaphragm and reduce the bright stars successively to different apparent magnitudes, or whether we use a series of different diaphragms and reduce them to the same apparent magnitude. Apparently there is no appreciable dependence of the slope of the scale upon the magnitudes to which the bright stars were reduced.

There is a point of interest relating to the systematic differences appearing in the deviations for the different groups which were referred to in the preceding paragraph. An important feature of the original reduction was the derivation of scales for the bright stars which should be homogeneous with the adopted scale for the fainter objects. The homogeneity obtained obviously refers not

merely to the general slope, but includes as well the irregularities of the assumed scale. But the scales actually found by the different combinations overlap in such a way as to produce at least a partial elimination of these irregularities, for each of the final magnitudes depends upon several faint stars differing considerably in brightness. The mean scale, therefore, should not only agree in slope with the assumed scale for the faint stars, but should be relatively free from any irregularities affecting the latter.

Turning again to the scales of Table IV, and noting that for each the magnitudes have all been determined by a comparison with a group of faint stars having approximately the same brightness, it appears that any irregularity in the scale for the faint stars will enter as a constant error. In other words, the zero point will not have been correctly determined, and this fact will reveal itself through the appearance of a constant systematic difference when a comparison is made with the mean scale, for, as has just been shown, the latter should be relatively free from irregularities.

TABLE V

IRREGULARITIES OF SCALE FOR FAINT STARS

Average Mag	Average Deviations		No. Determinations
	I	II	
10.2	−0m05	−0m05	9
11.0	− .02	− .03	49
11.5	− .02	− .02	57
12.0	+ .01	.00	68
12.5	+ .07	+ .07	49
13.0	− .01	+ .02	46
13.6	−0.02	−0.02	37

The average differences appended to the groups of Table IV, and collected in the second column of Table V, are therefore the relative errors of the zero points of the seven scales, and are to be interpreted as a measure of the deviations from uniformity in the scale for the faint stars.

Disregarding the first, which is of low weight, it is seen that the differences are all small, excepting that for magnitude 12.5. Here there seems to be an irregularity of appreciable amount. With

this exception, the internal consistency of the scale for the faint stars must be regarded as very satisfactory.

It is of interest to note that the result is largely independent of residual irregularities in the mean scale for the bright stars, due either to an imperfect elimination of any non-uniformity in the scale for the fainter objects, or to the uncertainties in the reduction constants for the screens and diaphragms. Broadly speaking, each of the scales of Table IV covers the entire range of the mean scale for the bright stars. Irregularities in the latter would therefore enter into all of the comparisons and affect by a constant amount the average deviations of Table V, so that the relative values would not be influenced thereby. The validity of this statement is easily tested, at least in so far as uncertainties in the reduction constants are concerned. As previously remarked, the average deviations in the last line of Table II do not, strictly speaking, represent the errors of the constants; but for a moment we may consider that such is rigorously the case. If, therefore, each of the deviations of Table II be corrected by the amount of the systematic deviation at the bottom of the column in which it stands, and if the results of Table V be revised accordingly, we shall obtain differences which are free from the errors of the reduction constants. The results of such a revision are given in the third column of the table. In only one case do these differ from those previously found by more than 0.01 mag. Finally, it may be remarked that the absence of any general progression in the differences of Table V is again an indication that the scales for bright and faint stars have sensibly the same slope.

There remains still to be effected a comparison of the scales of Table IV with that of Harvard. This is accomplished by adding to the mean magnitudes in Table IV the corresponding deviations in the last column and comparing the results with the Harvard magnitudes, or more directly, by adding the deviations of Table IV to the differences Mt. W.−H. in Table II. The weighted mean results for groups of stars are given in the third and fourth columns of Table VI. The stars included in each group are indicated in the second column, and the apparent magnitude to which they were reduced for comparison with faint stars in the first column. The

comparisons are shown graphically in Fig. 2. The divergence revealed by the original reduction appears in all cases excepting the

TABLE VI

COMPARISON OF INDEPENDENT SCALE DETERMINATIONS WITH THE HARVARD SCALE

APPARENT MAGNITUDE	STARS	MEANS		NO. DET.	DEV. FOR 1 MAG.
		Mag.	Mt. W.−H.		
9.9–10.6.....	1, 2*s*.....	4.97	−32	2	0m000
	3*s*, 6.....	6.48	−34	2	
	2*r*.....	7.52	−38	2	
	3*r*, 10.....	8.60	−30	3	
10.8–11.2.....	1.....	3.97	−56	3	
	2.....	4.78	−47	14	−0.071
	2*s*, 5, 3*s*.....	6.07	−38	13	
	7, 8.....	7.49	−30	5	
	4*r*, 11, 6*r*.....	9.33	−19	10	
11.3–11.7.....	1*s*.....	2.10	−62	7	−0.065
	3, 4.....	5.45	−34	25	
	6, 2*r*, 8.....	7.36	−39	8	
	9, 3*r*, 10.....	8.54	−25	12	
	13, 14, 7*r*.....	10.45	− 7	5	
11.8–12.2.....	1*s*.....	2.10	−58	7	−0.068
	1, 2.....	4.29	−51	5	
	2*s*, 5, 3*s*, 7.....	6.13	−36	27	
	10, 4*r*, 11.....	8.99	−25	6	
	12, 5*r*, 6*r*, 13....	9.96	+10	14	
12.3–12.7.....	3, 4.....	5.48	−37	9	−0.071
	3*s*, 6.....	6.42	−16	6	
	2*r*, 8.....	7.59	−11	11	
	9, 3*r*, 10, 11.....	8.68	−14	13	
	14, 7*r*.....	10.52	+ 6	5	
12.8–13.2.....	1*s*, 1.....	2.72	−57	6	−0.080
	2*s*, 5, 3*s*.....	6.08	−40	9	
	6, 7.....	6.95	−11	6	
	4*r*, 11, 12.....	9.45	− 3	10	
	5*r*, 6*r*, 13.....	9.99	0	11	
13.3–13.9.....	3, 4.....	5.48	−49	5	−0.085
	2*r*, 8.....	7.71	−32	11	
	9, 3*r*, 10, 4*r*.....	8.65	− 6	11	
	12, 13.....	9.94	−12	3	
	14, 7*r*.....	10.52	− 8	7	

first, and here the absence of the typical slope is probably due to the small number of determinations and a chance combination of large errors of observation. The divergence per magnitude for each of

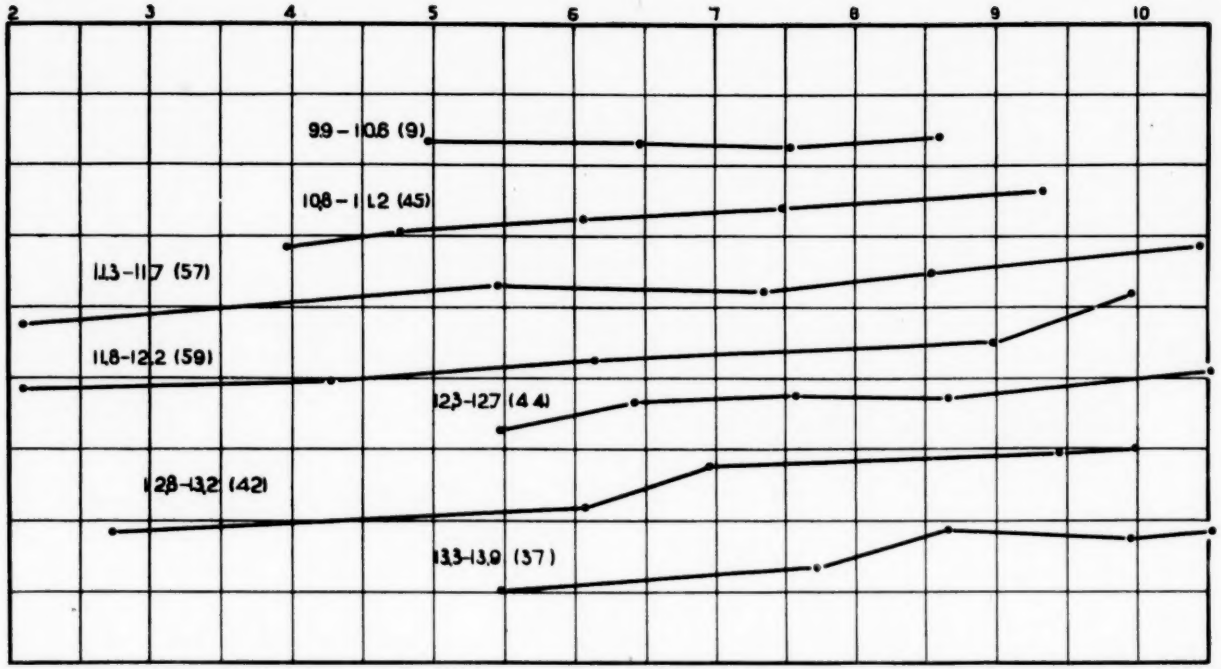

FIG. 2.—Comparison with Harvard (Mt. W.—H.) of separate scales derived by reducing bright stars to different apparent magnitudes. The limiting range of apparent magnitude and the number of observations are indicated at the left. The axis for each curve is the horizontal line immediately above. Vertical scale same as horizontal.

the scales is given in the last column of Table VI. Omitting the first, they show a high degree of accordance, and merely confirm the result previously found.

In addition to the results already described, the following may be mentioned as bearing upon the question of the scale for the brighter stars. Although these objects are in general so scattered that each has been separately compared by successive exposures with the faint stars surrounding the Pole, there are three, Nos. 15, 5, and 8, sufficiently close to each other to permit of their being photographed with a single exposure. These have been directly intercompared by means of a series of diaphragm exposures. The results are summarized in Table VII. Plate 764, for example, was

TABLE VII

RELATIVE MAGNITUDES OF STARS 15, 5, AND 8

PLATE NO	APERTURES	EXPOSURE TIME	STARS 15 AND 5		STARS 15 AND 8	
			Δm	Wt.	Δm	Wt.
P763....	14, 9, 6, 14+G, 9+G, 6+G	10^s	....	..	5.73	1
764....	14, 9, 6, 14+G, 9+G, 6+G	10, 30, 60	4.03	8	5.64	3
798....	14, 9, 6, 14+G, 9+G, 6+G	10, 30, 60	....	..	5.58	4
800....	32, 32+G, 32+G_2.....	10, 30	3.86	4	5.95	2
Mean Δm			3.97	..	5.69	..
Δm from Table II			3.89	..	5.68	..
Δm from *H.C.*, **170**			3.68	..	5.39	..

given three series of exposures of 10, 30, and 60 seconds, respectively. The apertures for each series were 14, 9, 6, 14+G, 9+G, and 6+G, having reduction constants of approximately 3, 4, 5, 6, 7, and 8 magnitudes. The images of any star for any series establish a scale which may be used for the determination of the relative brightness of the other two stars, the images for the latter being chosen from the same series. The resulting differences in magnitude for stars 15 and 5, and 15 and 8 are in the fourth and sixth columns of the table. Their means are 3.97 and 5.69 mags., respectively, while the corresponding differences from Table II are 3.89 and 5.68. The results, as far as the scale is concerned, are the same as before. The determination is of course not independent, for the same methods have been used for the reduction of the

light by a known amount as before. But the observations and reductions are quite different, and the results do not depend upon an intermediate comparison with the faint stars.

SUMMARY

1. The photographic magnitudes of the bright stars of the Polar Sequence have been determined by comparison with the stars included between 10.5 and 15.5. The reduction in the light was produced by combinations of screens and diaphragms whose absorption constants range from one to eleven magnitudes. The Harvard magnitudes for the faint stars, which agree closely with an independent determination, were used for the comparison. The methods employed should give a mean scale which is nearly homogeneous with that of the faint stars. Various tests indicate that this result has been realized with a reasonable approximation, and that the errors in the reduction constants affect the scale only to a minor degree.

2. Although the original program and the reduction involve merely an extension of the scale for the faint stars to include the brighter objects, it is possible to analyze the results into a series of separate and independent determinations. The zero points are fixed by comparison with groups of faint stars, but the slope depends wholly upon the reduction constants of the screens and diaphragms. The individual results are in close agreement with the mean scale.

3. A direct intercomparison of *Polaris* and two other bright stars having magnitudes 5.99 and 7.78, respectively, gave differences in brightness which are practically identical with those resulting from the mean scale.

4. An examination of the internal consistency of the Harvard scale for the faint stars from magnitude 9.9 to 13.7 reveals only minute deviations from uniformity.

5. A comparison of the mean scale for the bright stars with the corresponding Harvard magnitudes shows a clearly marked divergence, nearly linear in character, and approximately represented by the formula

$$\text{Mt. W.} - \text{H.} = -0^{m}070(11.0 - M),$$

which is valid for values of M from 2 to 9. The observed differences are zero at the tenth magnitude.

6. If the Mount Wilson magnitudes for the faint stars be included in the comparison, and if the results be referred to the international zero point, the differences from magnitudes 2 to 15.5 are expressed by

$$\text{Mt. W.}-\text{H.} = +0^{m}37 - 0^{m}070(11.0 - M)$$

in which the second term is to be disregarded for $M > 10$.

7. A comparison of the mean scale with the determinations of Schwarzschild and Parkhurst, which extend from magnitudes 4.0 to 7.5, gives sensibly constant differences which become zero when the Mount Wilson magnitudes are referred to the international zero point.

MOUNT WILSON SOLAR OBSERVATORY
February 27, 1913

DETERMINATION OF PERIODICITIES BY THE HARMONIC ANALYZER WITH AN APPLICATION TO THE SUN-SPOT CYCLE

By A. A. MICHELSON

It has been shown[1] that the values of the coefficients of a Fourier series may be obtained by a mechanical integration, with great facility and with considerable accuracy by the harmonic analyzer.

For this purpose the given function is copied on the machine, which then draws a curve whose ordinates at given distances along the axis of abscissas are proportional to the coefficients of the corresponding Fourier series.

The machine gives $C=\Sigma f(n)\cos n\theta$ or $S=\Sigma f(n)\sin n\theta$ which summations approximate to the corresponding integrals as the number of elements, m (in this case 80) increases.

Putting $\frac{m}{\pi}\theta=k$, the Fourier integrals are

$$C=\int_0^h f(x)\cos kx\,dx$$

$$S=\int_0^h f(x)\sin kx\,dx.$$

If the function to be investigated is periodic

$$f(x)=\Sigma a_n\cos(nx-\phi_n),$$

which gives

$$C_n=\frac{a_n}{\omega}[\cos\phi\sin\omega-\sin\phi(1-\cos\omega)]$$

$$S_n=\frac{a_n}{\omega}[\sin\phi\sin\omega+\cos\phi(1-\cos\omega)]$$

where $\omega=(k\pm n)h$.

In the Fourier integral h is infinite, and C and S are negligible except for $k=\pm n$.

[1] A. A. Michelson and S. W. Stratton, "A New Harmonic Analyzer," *Am. Jour. Sci.*, (4) **5**, 1, 1898.

The intensity of the "periodogram"[1] is $I=C^2+S^2$ which gives in the present instance $I_n=a_n^2\dfrac{\sin^2\frac{\omega}{2}}{\left(\frac{\omega}{2}\right)^2}$ which has its maximum value a_n^2 at $\omega=0$. The frequency is the value of k at this point. The

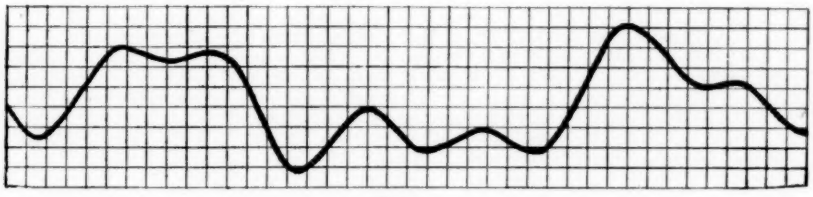

FIG. 1

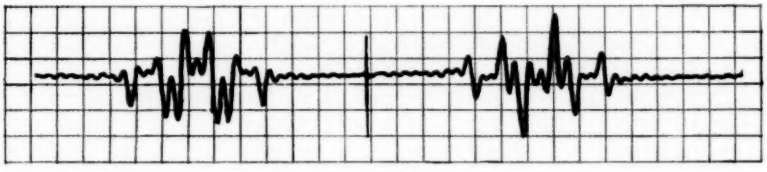

FIG. 2

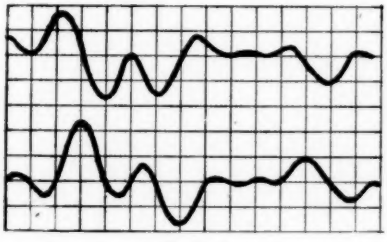

FIG. 3

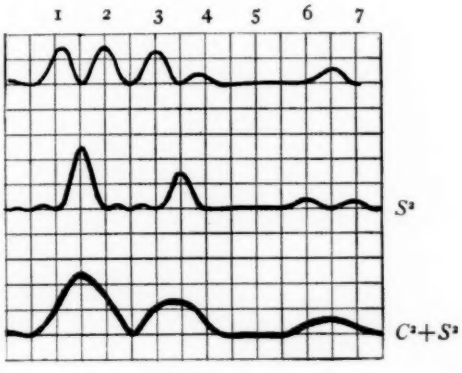

FIG. 4

maximum value of the curve given by the harmonic analyzer is therefore proportional to the amplitude a_n of the corresponding simple harmonic element of the function.

[1] Arthur Schuster, "The Periodogram and Its Optical Analogy," *Proc. Royal Society*, **77**, 136, 1906.

The phase is also given by $\tan \phi_n = \frac{S_n}{C_n}$ at this point.

A test of the performance of the machine was made in the analysis of the function Y whose graph is given in Fig. 1.

The C and S curves are given in Fig. 2, and the same on a larger scale, together with C^2, S^2, and C^2+S^2 in Figs. 3 and 4.

From these we find the values given in the second column, while the first gives the elements from which Y was constructed. The agreement is quite as good as could be expected.

$Y = -3.0 \cos (1.5x - .5\pi)$	$Y = -2.9 \cos (1.5x - .45\pi)$
$+2.0 \cos (3.5x - .5\pi)$	$+2.0 \cos (3.4x - .40\pi)$
$+1.2 \cos (6.3x \quad)$	$+1.3 \cos (6.4x - .05\pi)$

It may be noted that the method fails, and for the same reason as in the "spectroscopic" method, when the components are too close together. In fact the "limit of resolution" is the reciprocal of double the number of periods N over which the integration extends. Thus if there are two components of 9.5 and 10 periods respectively in this distance, the limit of resolution $\frac{\delta T}{T} = \frac{1}{20} = \frac{1}{2N}$, and the two components are distinctly resolved.

A similar limitation occurs when there is but one period or less in the total length—for then the two values corresponding to $k = \pm n$ partially overlap.

A similar treatment of the sun-spot curve, as furnished in the interesting paper of H. Kimura[1] gave the following results:

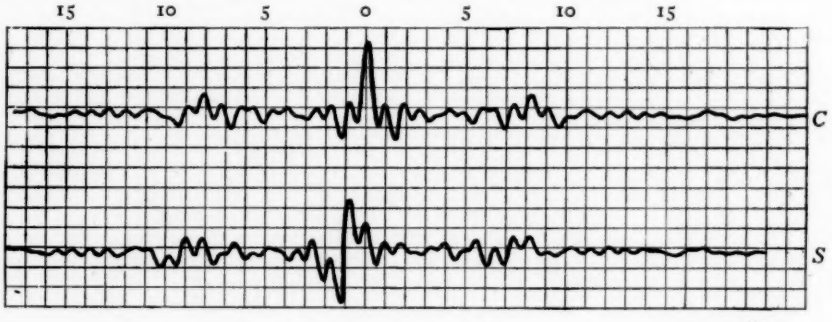

FIG. 5 N divs. $= \frac{80}{N}$ yrs.

[1] *Monthly Notices, R.A.S.*, **73**, 543, May 1913.

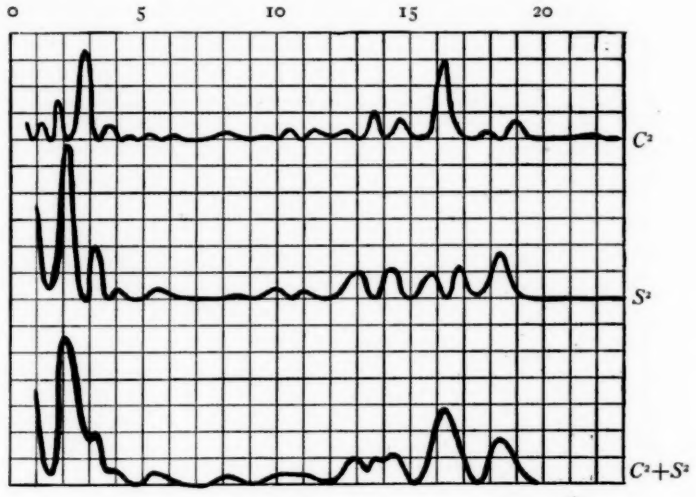

FIG. 6 N divs. $= \frac{160}{N}$ yrs.

The curve was first taken from 1750 to 1850 and furnished the C and S curves given in Fig. 5. From these the "intensity" curve C^2+S^2 was found and reproduced in Fig. 6.

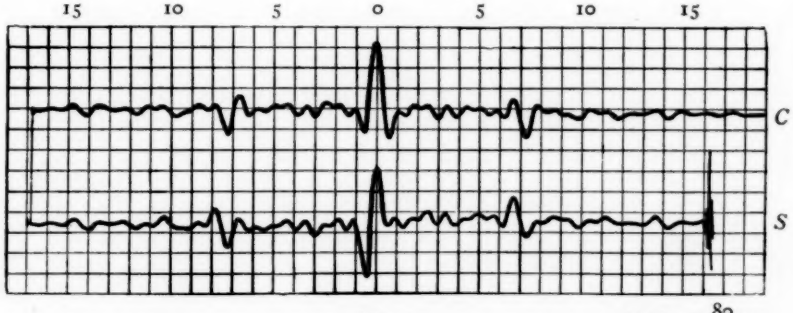

FIG. 7 N divs. $= \frac{80}{N}$ yrs.

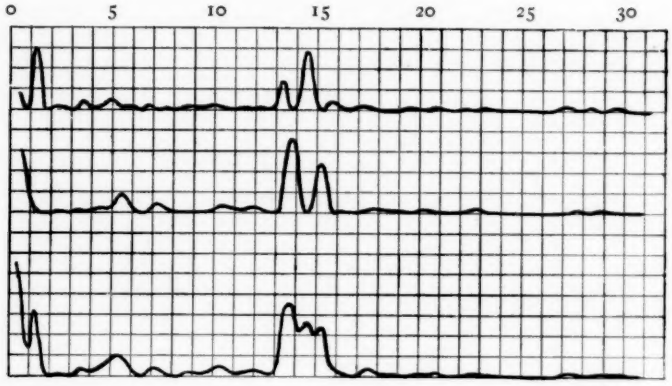

FIG. 8 N divs. $= \frac{160}{N}$ yrs.

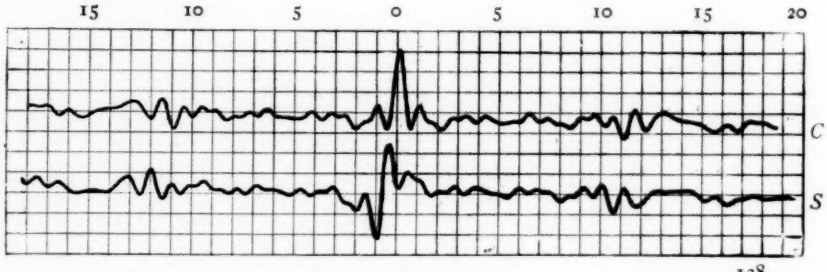

FIG. 9 N divs. $= \frac{128}{N}$ yrs.

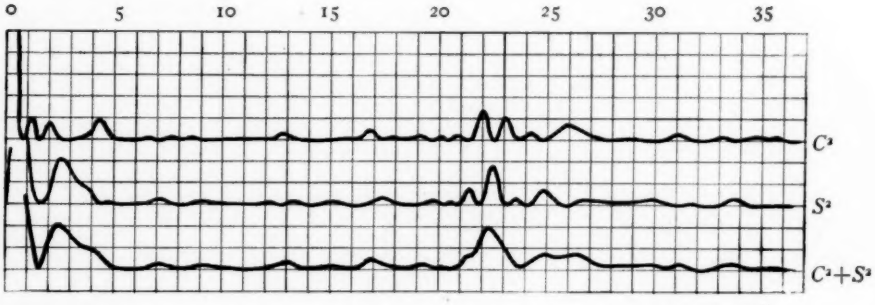

FIG. 10 N divs. $= \frac{256}{N}$ yrs.

A similar treatment of the interval from 1800 to 1900 gave Figs. 7 and 8, while the entire curve from 1750 to 1910 furnished results given in Figs. 9 and 10.

The periods and amplitudes deduced from these curves are given in the following table:

1750 to 1850		1800 to 1900		1750 to 1910	
Period	Amplitude	Period	Amplitude	Period	Amplitude
80 yrs.	5.3	129 yrs.	3.9	105 yrs.	3.5
50 "	2.0	44 "	1.4	58 "	2.2
40 "	1.4	30 "	2.2	36 "	1.3
29 "	1.6	22 "	1.4	28.5 "	1.0
19.5 "	1.2	18.2 "	1.0	19.9 "	1.3
15.7 "	1.4	15.4 "	1.6	15.1 "	1.5
14.5 "	1.4	13.3 "	1.1	13.3 "	1.3
12.5 "	2.2	11.6 "	4.2	11.4 "	2.9
11.2 "	2.4	11.0 "	3.6	10.3 "	1.8
9.9 "	3.8	10.5 "	3.5	8.6 "	1.8
8.7 "	2.9	9.1 "	1.4	8.3 "	1.6

It appears on inspection of these tables that there is but little relationship between the two parts of the sun-spot record, notwithstanding the fact that one-half is common to both—except for the 11-year period; and even here this result in the first case is quite dubious, while in the second it is much more probable that there are three periods (of 11.6, 11.0, and 10.5). The third or complete 160-year record shows a fairly isolated period of 11.4 years.

The given curves do not extend beyond the 6-year period, but in another case they have been extended to something less than the 3-year period with no certain indications of anything like a component having a period of 8.36 years, or of 4.79 years as found by Schuster.

Indeed it would seem that with the exception of the 11-year period and possibly a very long period (of the order of 100 years), the many periods found by previous investigators are illusory.

It will probably be found that even the 11-year period is in fact not constant, but is subject to secular change; in which case the sun-spot curve should be represented, not by a summation of homogeneous elements, but by an expresson of the form $Y=\psi(t)\cos[nt+\phi(t)]$.

NOTE

In a recent paper, C. V. Burton[1] suggests a method not essentially different from that given by Lord Rayleigh[2] for the resolution of a function by the diffraction of light by means of a screen whose apertures permit light to pass whose amplitude is proportional to the function to be analyzed.

It can readily be shown that the resolution can also be effected by a diffraction grating whose errors of spacing are proportional to the function—provided these errors are not too large compared with the average grating space. The periodicities in the function will be presented in the form of "ghosts" about the principal spectral line.

[1] *Proc. Phys. Soc.*, June 1913.

[2] *Phil. Mag.*, (6) **5**, 238, 1903.

A PROBABLE PARALLAX OF THE NORTH AMERICA NEBULA

By E. BUCH ANDERSEN

Till now it has been possible only in a few cases to determine parallaxes of nebulae in a direct way and only for those nebulae which have been proved to be in physical connection with one or more fixed stars. All that can be said with any certainty is that the gaseous nebulae in general do not differ from the fixed stars in regard to either proper motion or radial velocity, so that gradually it has come to be regarded as probable that most of the gaseous nebulae belong to our stellar system. Apart from the ordinary physical connection between nebulae and stars, of which several examples have been noticed (*Pleiades* nebulae, *Orion* nebula, the annular nebula in *Lyra*, etc.), the vast irregular gaseous nebulae often seem to have a peculiar influence on the stars in their vicinity; their density may suddenly decrease to such a degree that it looks as if the nebula were placed in a dark hole. As characteristic examples may be mentioned: *Orion* nebula, nebula about ρ *Ophiuchi*, ξ *Persei*, North America nebula, γ *Scuti*, 15 *Monocerotis*, and several others.[1] It is a curious fact that all these objects are found in regions very rich in stars (with the exception of the *Orion* nebula all are in the Milky Way itself); perhaps it is accidental, because the phenomena can be observed only in places where the star-density is great; or perhaps it is the real distribution and another proof that these nebulae belong to the stellar system. Such dark regions may also be found without apparently containing any nebula (e.g., near α *Cephei*, δ *Ophiuchi*, η *Carinae*, etc.), which, however, are in the Milky Way.

As to the real cause of these "rifts" there are various opinions, two of which have held their own. According to the first the nebula has moved and in some way absorbed or destroyed the stars which it met on its way, so that the "rift" must be considered as a tube or a canal, through which the nebula has passed. The other explanation is that the luminous nebula is sometimes surrounded

[1] See, e.g., *Astrophysical Journal*, **31**, 8, 1910.

by a dark nebula or accumulation of matter which completely absorbs the light from the stars behind it and so hides them from us. The first theory is supported by the facts that the luminous nebula often lies in one end of the "rift," and that the star-density, which in the immediate proximity of the nebula suddenly decreases, in the nebula itself increases again, even if it does not reach its normal value. The fact, however, that the dark canals sometimes reach into the luminous nebula and both here and outside are far branching, and that the "rifts" are often absolutely free from stars except a few bright ones, which are evidently in front of it, is in favor of the second. Anyhow it is rather improbable that the nebula should be of such enormous size that it should destroy the stars altogether to the uttermost limits of the system, because this must be considered to have its greatest extent in the very direction in which these nebulae are observed.

Perhaps both theories separately are correct, so that it is necessary in a single case to decide which influence is the active one. It is, however, obvious that, whatever the conditions may be in the "rifts," the stars which are between the nebula and us must be unaffected by the phenomena. It may be possible to make use of this fact to get an approximate idea of the distance of the nebula, and in the following, this has been tried for the North America nebula.

In 1902 an investigation into the star-density in the vicinity of the North America nebula was published by A. Kopff.[1] The region in question was photographed by means of the 16-inch objective *a* of the Bruce telescope and with an exposure of about 5^h. The plate was divided into small squares, whose sides had the direction N.–S. and E.–W., and the stars contained in each of them were counted. The results obtained in this way are given in a table together with the co-ordinates for the central point of each square, but without any information as to how far the stars indicated on the plate diminish in brightness. The peculiar distribution of the stars, easily observable on the photographs themselves, is verified by this investigation.

In the following table I have set forth some of the figures given in Kopff's work for two zones, which extend across the North America nebula in direction E.–W., lying respectively between $+43°$ to $44°$ and $+44°$ to $45°$ N. Decl. The number of stars in

[1] *Publ. Astroph. Inst. Heidelberg*, 1, 181, 1902.

each zone is stated with respect to small squares, of which one side (N.–S.) is 1°, while the other is the same as in Kopff's original squares. The first column in the table gives the right ascension for the central point of this.

TABLE I

δ =	43°–44°	44°–45°	δ =	43°–44°	44°–45°	δ =	43°–44°	44°–45°
α	*n*	*n*	α	*n*	*n*	α	*n*	*n*
20^h46^m2	90	144	20^h53^m3	148	590	21^h0^m4	141	598
47.0	64	122	54.1	232	532	1.2	184	470
47.8	85	114	54.9	337	626	2.0	299	373
48.6	75	198	55.6	339	604	2.7	526	313
49.4	80	326	56.4	342	634	3.5	625	344
50.2	83	431	57.2	314	603	4.3	718	567
50.9	95	538	58.0	280	738	5.1	728	738
51.7	135	562	58.8	247	721	5.9	775	1036
52.5	131	527	59.6	162	722	.....	...	....

For the same two zones I have next by means of the ***Bonner Durchmusterung*** examined the distribution of stars according to their brightness. Owing to the very small number of stars which could be taken into consideration by this investigation, it was necessary to make the E.–W. side of the squares somewhat larger. In the following table the results are given; the right ascension again refers to the central point of the sides in question.

TABLE II

α	δ = 43°–44°					α	δ = 44°–45°				
	br. th. 6^m5	6^m5-7^m4	7^m5-8^m4	8^m5-9^m4	9^m5		br. th. 6^m5	6^m5-7^m4	7^m5-8^m4	8^m5-9^m4	9^m5
20^h42^m	0	1	1	3	2	20^h42^m	0	0	0	12	4
44	0	1	1	5	5	44	0	0	1	6	3
46	0	0	1	5	1	46	0	0	0	11	2
48	1	0	1	7	3	48	1	0	3	4	6
50	0	1	2	1	3	50	0	1	2	8	2
52	0	1	1	3	4	52	0	1	3	9	3
54	1	2	1	0	3	54	0	1	1	7	5
56	0	1	1	0	6	56	0	0	0	7	4
58	0	0	1	3	3	58	0	1	2	5	2
21 0	1	0	1	1	5	21 0	0	0	2	5	6
2	0	0	2	4	4	2	0	0	3	5	3
4	0	1	3	4	2	4	0	1	2	3	2
6	0	0	1	7	8	6	0	0	1	8	7
8	0	1	3	6	3	8	0	0	2	7	4

To show better the distribution of the stars, the foregoing tabulation is given graphically in Fig. 1 and Fig. 2 for the two zones, with the right ascensions as abscissae; the ordinate for Kopff's enumeration has been divided by 10.

The lowest curve, which is derived from a very large number of stars, represents the real law of distribution, and it is evident that the stars, which are at the same distance as the nebula or

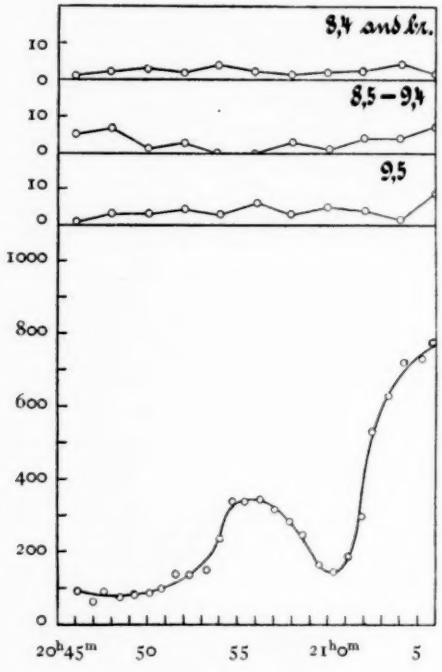

FIG. 1

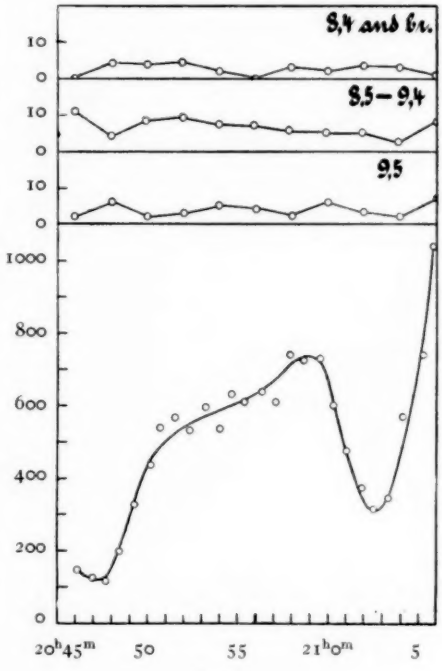

FIG. 2

behind it, must follow this law, whereas stars lying in front must be independent of it. From the drawings it appears clearly that stars of $8^m.5$ and smaller ones follow the law of distribution such as Kopff has found with the well marked minima near 20^h47^m and 21^h3^m, while the brighter stars have absolutely no tendency to decrease in number in these places.

In accordance with the above the parallax of the North America nebula must lie near that of stars of $8^m.5$, which, when no regard

is paid to proper motion, according to Kapteyn, has the probable value of 0".007. As matters now stand, this result fits in well with other parallaxes of nebulae. Kapteyn has from the proper motion derived the mean parallax of the gaseous nebulae and found it to be 0".005, while Newkirk for the central star of the annular nebula of *Lyra* has found 0".07.

As Kopff's investigations extend over a comparatively small area, it has been impossible to follow the curves out to both sides, and find any further correspondence; but in consideration of the small amount of material at our disposal for finding the distribution of bright stars on an area of that size, it is not surprising that the single points of the curves do not lie more continually or that these do not show a more definite shape. Finally for the stars of $9^m.5$ it must be remembered that the *B.D.* at this stage of brightness can lay no claim to completeness.

As mentioned above and as may be seen, too, from the drawings, the density of stars will increase in the luminous nebula. This was said in support of the theory that the surrounding dark region is a real hole and not due to the presence of dark nebulous matter; we must suppose that the nebula is most luminous where it has its greatest density as seen from the earth, and where it must consequently absorb the greater part of the light from the stars behind it.

This may be only a hypothesis. The absorption is less dependent on the density than on its constitution, analogous to conditions connected with molecular and colloid dispersions. If, e.g., mastic is dissolved in methylic alcohol, in which it is easily soluble, we get a clear, transparent, molecular-dispersed solution; when, however, a little of this is poured into a large amount of water, the whole will immediately become milky and opaque; it has turned into a colloid solution of mastic in water, which absorbs much more light than the molecular solution, although it contains a much smaller part of matter per unit of volume.

The whole matter is a question of the diameters of the particles, which are suspended in the solution. If they are of such a magnitude that they are able to divert the light, the solution will show the well known Tyndall phenomenon, the particles will scatter the entering light in all directions and let only a little of it pass through.

The same theory may be applied to the nebulae. It is in this connection non-essential what process is the real cause of the luminosity of the nebulae. From the spectra it appears that the nebula in the more luminous parts is molecular, i.e., gaseous, and that it on the contrary does not send out any light from the parts where the nebulous matter eventually is not gaseous. In the few cases where it has been proved that a nebula (especially a spiral nebula) has both a gaseous spectrum and a continuous spectrum, we may assume with a fair amount of certainty that this originates in matter in solid or liquid form and that it is radiation due to temperature. It is very unlikely that such should be the case with the emission of the nebula itself.

In a previous work[1] E. E. Barnard puts the question, What will be the fate of the nebulae "when they die out; for since it does not seem any longer necessary to use these vast bodies of gaseous matter for the making of suns, the dying-out of nebulae is a probability fully as warranted as the belief and certainty that the stars must die out." Perhaps the dark parts may be considered as nebulous matter, which is no longer able to emit light. At any rate it is not unnatural to suppose that the molecules of the gaseous nebula in the outer regions for certain reasons, e.g., temperature, will join together, forming greater particles which do not emit any light themselves but to a certain degree absorb and divert light coming from without. Whether such particles have the structure of dust, solid air, or the like, the nebula in these parts will show a considerably greater absorption, although the absolute density may be far less than in the more luminous part. A nebula of this structure will by intense irradiation show the Tyndall phenomenon, and something of that kind has probably been observed in *Nova Persei*.

Sometimes the star-density is seen to be very variable in the more luminous parts of the nebula too; this may probably originate in the presence of dark matter behind. Surely interesting information about the physical constitution of the gaseous nebulae may be had by observing the light-curves in light of both short and long wave-lengths for variable stars, which are behind a nebula, and whose light must pass through it, and then comparing the results

[1] *Astrophysical Journal*, **25**, 218, 1907.

with such as are obtained from regions as free from nebula as possible. The effect on light passing through an air-mass of the same size as the North America nebula (computed from the parallax mentioned above) will be a shift between red and violet rays of several minutes, even if the average density of the air is only a fraction of an atmosphere.

As mentioned before, this particular dependence of the star-density on a nebula appears by no means in all nebulae. Objects like the *Andromeda* nebula, etc., with a continuous spectrum show no effect on the stars in the vicinity. The phenomenon seems to appear only in gaseous nebulae and mostly together with the great irregular nebulae which lie in or near the Milky Way. Whether any attention is to be paid to this latter fact cannot yet be decided; the phenomenon may be present even if its extension or the star-density in the vicinity is so inconsiderable that we are not able to observe it.

It may with certainty be said that nebulae which are observed in connection with such characteristic dark regions (whatever their real nature may be) belong to our stellar system, and by using the method described in this paper for other objects as well, perhaps some results can be gained which will have value by being more than merely hypothetical.

URANIA OBSERVATORY
COPENHAGEN
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THE SPECTRA OF MERCURY IN THE SCHUMANN REGION

By THEODORE LYMAN

The spectra of mercury in the Schumann region are of some interest at the present time, partly because of their bearing on the subject of series relations and partly because the mercury arc in quartz has been recently extensively employed in photo-chemical, biochemical, and photo-electric researches.

Very little is known of the spectra in the region on the more refrangible side of λ 1900. The work of Huff[1] has carried the Geissler-tube spectrum to λ 1872.2. A single line in this spectrum near λ 1850 has long been familiar to me and was observed by Handke[2] in 1909. The spectrum of the mercury arc in quartz has been investigated by several observers. Hughes[3] found a strong line near λ 1850 and Tian,[4] working with a vacuum prism spectroscope, corroborated the observation, while, with a spectroscope in air, he found lines at λ 1846 and λ 1848.[5] Henri,[6] Fabry and Buisson,[7] and others have investigated the connection between the radiation and the electric conditions of the arc.

As my vacuum grating spectroscope is better adapted to the investigation of the Schumann region than the instruments used by the investigators just mentioned, it seemed worth while to take up the subject.

The arrangement for studying the spark spectrum was exactly similar to that employed in my work on the spectra of the alkali earths.[8] A strontium amalgam, a barium amalgam, and a globule of mercury were used successively as terminals. The spark was in an atmosphere of hydrogen. The spectra obtained with the three substances are practically identical. They are characterized by

[1] Kayser, *Handbuch*, **5**, p. 548.

[2] Inaug. Diss., Berlin, 1909.

[3] *Proc. Camb. Phil. Soc.*, **16**, 428, 1912.

[4] *Comptes rendus*, **155**, 141, 1912.

[5] *Ibid.*, **152**, 1483, 1911.

[6] *Ibid.*, **153**, 426, 1911.

[7] *Ibid.*, **153**, 93, 1911.

[8] *Astrophysical Journal*, **35**, 344, 1912.

numerous lines extending from λ 1876 to the limit set by the transparency of fluorite. All the strong lines lie between λ 1876 and 1650. The lines do not present any very obvious regularity of arrangement.

Two forms of mercury arc were employed. In one, the whole vessel was of quartz, and the light entered the spectroscope through the side of the lamp. In the other, the vessel was of glass fitted with a fluorite window. The spectrum is dominated by the broad unsymmetrical line at λ 1849.6, which considerably exceeds in strength any single line in this region in the spectrum of any substance with which I am acquainted. The line is easily reversed, a fact which has already been noted by Tian.[1] The other six lines which go to make up the arc spectrum, though much inferior in intensity, are still fairly strong.

The relation between the spark and arc spectra is what one would expect from the behavior of mercury in the visible and ultraviolet, for the spark spectrum is rich in lines and the arc spectrum contains but few.

From the point of view of theory, it is interesting to note that the lines at λ 1849.6 and λ 1402.5 in the arc are the two first members of the main series predicted by Paschen;[2] the third member, which should occur at λ 1268.9, is within the limit of the transparency of fluorite, but I have not been able to find it. This was to be expected, for, as the intensity falls off very rapidly between λ 1849.6 and λ 1402.5, the next line in the series must be very faint indeed. The members of the second series,[3] λ 1435.6, 1307.8, 1259.3, and 1235.9, cannot be found in the arc, though 1307.8 occurs faintly in the spark. This does not necessarily show that the prediction is wrong, for experimental difficulties render the detection of faint lines beyond λ 1350 almost impossible.

Hughes[4] predicted the nature of the spectrum of the mercury arc in the Schumann region, basing his conclusions on photoelectric data. Later experiments[5] have somewhat changed his numerical values. Neither the original prediction nor the corrected data agree with the spectrum as I have found it.

[1] *Comptes rendus*, **155**, 141, 1912.
[2] *Ann. d. phys.*, **35**, 860, 1911.
[3] *Ibid.*, **40**, 605, 1913.
[4] *Phil. Mag.*, **21**, 393, 1911.
[5] *Phil. Trans. Roy. Soc. Lond.*, **212**, 205, 1912.

Following my usual practice, the observations which have been recorded were obtained after the spectroscope had been repeatedly exhausted and washed with dry hydrogen. If a photograph was taken with air in the spectroscope, the appearance of the line at λ 1849.6 was profoundly modified. The broad line, which often extended over thirty angstroms, was now replaced by three groups of rather faint, sharp lines. This is, undoubtedly, the phenomenon observed by Steubing,[1] and this is the phenomenon which all investigators will observe who work with an apparatus in which the light from the mercury arc must traverse a considerable air-path before falling on the photographic plate. It is possible that some of Tian's measurements refer to the strongest of these lines or bands. Except for these lines, there is nothing visible in the mercury arc below λ 1900 when investigated through air.

Steubing has attributed the lines to the fluorescence of oxygen, and their narrowness and sharpness lend color to this interpretation. It is not impossible, however, that the effect is produced by the absorption of the air after the light has passed through the slit in the spectroscope. Absorption must enter into the phenomenon, even if Steubing's view is correct.

When the mercury arc in quartz is employed in photo-chemical, biochemical, and photo-electric experiments, it is obvious that very different results may be expected, if the light has to traverse a short air-path, from those which will result if the air-path is long. In the first case, the full energy of the great line at λ 1849.6 will be effective; in the second, only the feeble action of Steubing's lines will be felt. It is well for workers in these fields to bear this fact in mind. Experiments show that biochemical, like photo-electric action, increases rapidly with decrease in wave-length. It is, therefore, this line λ 1849.6 which, of all lines in the mercury spectrum, is the most active in producing *abiotic* effects when the organisms under observation can be brought into close proximity to the lamp. In this connection, one must remember that even fused quartz is sufficiently transparent to transmit this radiation strongly. On the other hand, it must not be forgotten that water in thickness of even one millimeter is very opaque at λ 1850.[2] The scarcity and

[1] *Ann. de phys.*, **33**, 572, 1910.

[2] *Nature*, **84**, 71, 1910.

feeble nature of the other lines in the Schumann region render the mercury arc inefficient in producing effects which depend on shorter wave-lengths. For example, the light from the mercury arc produces far less volume ionization than results from the use of a hydrogen tube.

A detailed account of the experiments follows. The spark chamber and the manner in which it was attached to the spectroscope is shown on p. 344 of the *Astrophysical Journal*, **35**, No. 5. All the apparatus and the experimental procedure were exactly as there described. The exposures were usually 20 minutes.

The strontium amalgam contained about 75 per cent of mercury. The barium amalgam contained about 80 per cent of mercury. When pure mercury itself was employed, it was held in a steel cup; the upper electrode was a point of soft steel. As has already been observed, the three spectra obtained in this way are nearly identical. The spectrum obtained with pure mercury, however, was quite feeble in the more refrangible end of the spectrum; in fact, beyond λ 1750 all the lines are very faint and the group beyond λ 1350 cannot be seen at all. This is probably due to the absorption of the metallic vapor round the spark. That the lines between λ 1750 and λ 1260 are due to mercury itself, is proved by the behavior of the amalgams. For, though the strontium amalgam was of rather doubtful character, the barium amalgam was known to be pure. The spectra of these two substances are identical; it seems probable, therefore, that their common spectrum is due to their common ingredient, namely, mercury.

Table I contains the wave-lengths in vacuum with the frequencies added for the convenience of those who may be interested in computations connected with series spectra. The line λ 1849.6, so strong in the arc, appears in the amalgam spectrum as a faint band; it is reversed in the spark from mercury itself. Of the remaining six arc lines, only two surely appear in the spark. The values in the table have been compared with those for aluminum, iron, and the alkali earths, and I hope most of the impurities have been eliminated. The measurements were made by the method of shifted spectra.[1] The absolute position of the lines should not be

[1] *Astrophysical Journal*, **23**, 202, 1906.

TABLE I

Spark			Arc		
λ	I		λ	I	1/λ
1269.7	5		.....	.....	78758
1277.1	4		.....	.....	78302
1280.7	4		.....	.....	78082
1305.6	3	?	.....	.....	76593
1307.9	2		.....	.....	76458
1321.4	3		.....	.....	75677
1323.2	4		.....	.....	75574
1326.4	4		.....	.....	75392
1330.8	7	*Ba?*	.....	.....	75143
1335.4	5		.....	.....	74884
1350.4	3		.....	.....	74052
1361.0	4	*Ba?*	.....	.....	73475
1378.0	1		.....	.....	72569
1379.0	1		.....	.....	72516
1400.4	2		.....	.....	71408
			1402.5	6	71301
1404.3	1		.....	.....	71209
1414.4	4		.....	.....	70701
1416.9	4	*Ba?*	.....	.....	70576
1481.6	1		.....	.....	67494
1495.0	3		.....	.....	66889
			1518.6	3	65850
1527.4	5		.....	.....	65470
1540.5	2		.....	.....	64914
1548.4	1	?	.....	.....	64583
1550.8	2	?	.....	.....	64483
1561.0	1	?	.....	.....	64061
1568.0	2		.....	.....	63775
1570.3	1		.....	.....	63682
1592.9	8	?	.....	.....	62778
1599.4	7		.....	.....	62523
1641.5	1		.....	.....	60919
1647.4	9		.....	.....	60702
1649.8	10	*br*	1649.8	8	60613
1652.5	3		.....	.....	60514
1654.7	5		.....	.....	60434
1656.9	3	?	.....	.....	60353
1662.6	7		.....	.....	60146
1670.1	1		.....	.....	59876
1671.0	7		.....	.....	59844
1672.5	4		.....	.....	59790
1677.9	10		.....	.....	59598
1681.6	1		.....	.....	59467
1692.7	2		.....	.....	59077
1695.0	2		.....	.....	58997
1716.6	1		.....	.....	58254
1718.1	2		.....	.....	58204
1720.7	2		.....	.....	58116
1724.2	1		.....	.....	57997
1726.8	1		.....	.....	57911
1735.8	1		.....	.....	57610
1738.3	8		.....	.....	57527
1740.2	6		.....	.....	57464
1741.0	2		.....	.....	57438

TABLE I—*Continued*

Spark		Arc		
λ	I	λ	I	1/λ
1742.7	2	.....	.....	57382
1745.2	1	.....	.....	57300
1751.5	1	.....	.....	57094
1756.0	2	.....	.....	56947
1759.7	6	.....	.....	56828
1764.0	1	.....	.....	56689
1770.1	1	.....	.....	56494
		1774.9	5	56341
1775.2	3	.....	.....	56331
1780.0	1	.....	.....	56179
1787.7	1	.....	.....	55938
1792.7	5	.....	.....	55781
1796.2	7	.....	.....	55673
1798.7	9	.....	.....	55595
1800.7	1	.....	.....	55534
1803.9	3	.....	.....	55435
1806.5	2	.....	.....	55355
1808.3	2	.....	.....	55300
		1810	3 band	55248
1816.1	1	.....	.....	55063
1820.8	10	.....	.....	54921
1823.8	3	.....	.....	54830
1826.2	4	.....	.....	54758
1832.7	6	1832.6	4	54564
1837.1	1	.....	.....	54433
1840.8	7	.....	.....	54324
1849.0	5	.....	.....	54083
1849.6	5 reversed	1849.6	>100	54065
1850.3	3	.....	.....	54045
1853.4	4	.....	.....	53954
1859.5	2 *br*	.....	.....	53778
1861.0	1	.....	.....	53734
1862.3	1	.....	.....	53697
1865.1	1	.....	.....	53616
1869.4	10 *br*	.....	.....	53493
1875.7	6	.....	.....	53313

in error by more than three-tenths of an angstrom unit. The relative distance between neighboring lines, which involves the setting error only, should be accurate to about one-tenth of a unit.

My work on the mercury spectrum from a vacuum tube in the region below λ 1900 has been carried on only at pressures under 3 or 4 mm and has been mostly incidental to the study of gas spectra. I am able to recognize but one line, that at λ 1849.6. Handke[1] gives the line λ 1850.0. The difference of four-tenths of a unit between his value and mine is not confined to this particular line.

[1] *Op. cit.*

All his measurements in this region give results about half a unit higher than mine.

In my work on the arc in quartz, two lamps were employed; they were both of the form shown in Fig. 1. They were constructed of tubing 1 cm in external diameter; the portion between the electrodes was 13 cm long. The drawing is very nearly to scale.

In one of the lamps, on the side of the tube at A, a small bulb was blown. This bulb was then flattened, as well as might be, and formed a rough window. When the exposure was to be made, the

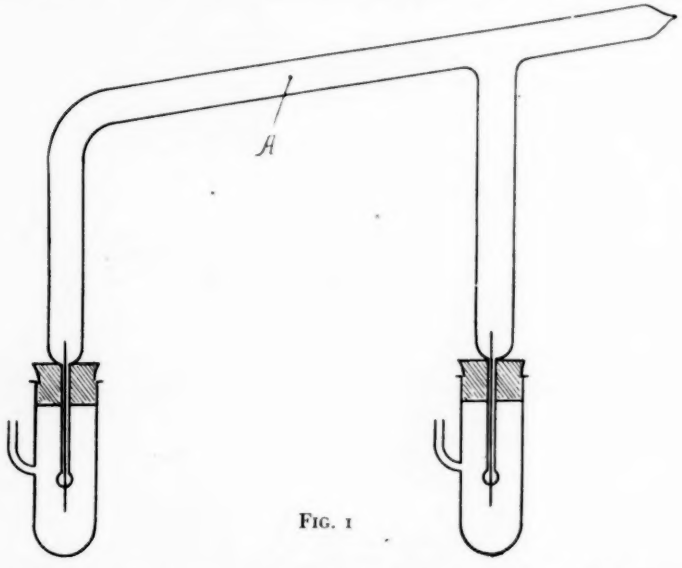


FIG. 1

flattened portion of the lamp was pressed against the fluorite window of the spectroscope, the free air-path being of the order of two-tenths of a millimeter. The other lamp had no bulb; the tube was pressed directly against the spectroscope window. Both lamps yielded the same results. The lamps were exhausted in the usual manner and then sealed; they were started by tipping. A current of 1.5 amperes was employed; the drop across the terminals was 80 volts. The exposure was usually about five minutes and was always terminated either by the cracking of the fluorite window or by the melting of the cement which held it in place on the spectroscope.

The line λ 1774.9 is the most refrangible radiation on these plates. The fact that the spectrum terminates at this point is probably due to two causes: first, the absorption of the quartz, and second, the absorption of the dense mercury vapor itself. The broad line or band at λ 1849.6 is always unsymmetrical, the maximum of intensity lying well toward the more refrangible side. With the quartz lamp, the band is always strongly reversed; the width of the whole band is usually about 30 or 40 units; the reversed portion is about 6 units wide.

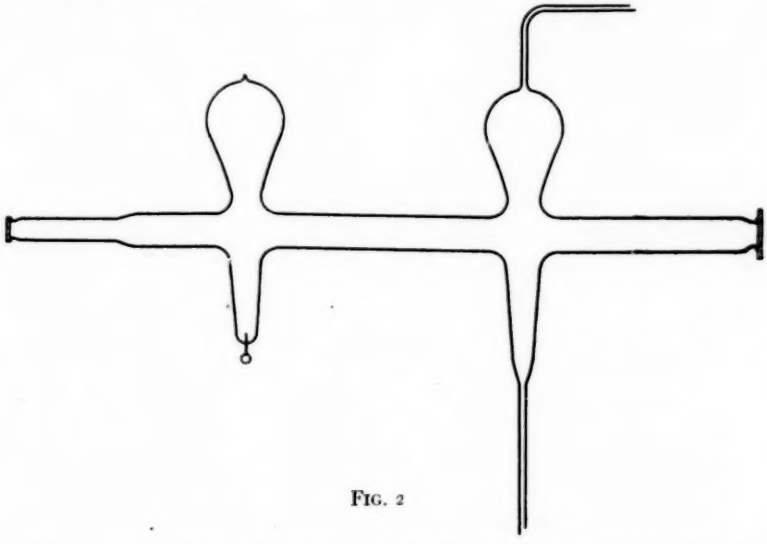


FIG. 2

The second type of lamp was designed to do away with the absorption of quartz and to enable experiments to be made with the mercury vapor at low pressures. With it, the line λ 1849.6 could be obtained unreversed, and with it, the lines below λ 1775 were discovered. It is illustrated in Fig. 2. The end next the spectroscope was closed by a fluorite plate attached with Khotinski cement and was 24 cm from the nearest mercury electrode. In the final arrangement, the end was fitted into a brass cone (not shown in the sketch) which, in turn, made an air-tight joint with a cup attached to the face plate of the spectroscope, the arrangement being exactly that illustrated on p. 102 of Vol. **33** of this *Journal*. Thus the light from the mercury lamp passed through only one fluorite window

about 1 mm thick before entering the spectroscope. The arc was started by raising the mercury in the barometer column which formed one of the electrodes. During an exposure, the lamp was always connected to a powerful Trimount mechanical pump; the pressure, registered on a McLeod gauge situated midway between lamp and pump, was always less than one one-hundredth of a millimeter.

Though the arrangement was simple yet the experiments proved difficult and tedious. This was mainly due to the formation of opaque films on the fluorite window, which always necessitated breaking the air-tight joint between the lamp and the spectroscope and sometimes even required the removal of the window from the lamp itself. Because of the danger to the spectroscope from mercury vapor, the window could never be discarded altogether.

The lamp took 4 amperes with 25 volts drop in potential across the terminals. The time of exposure necessary to bring out the fainter lines in the spectrum was about fifteen minutes. A record of the line at λ 1849.6 could be obtained with ten seconds' exposure, however.

Even with this form of lamp, the line at λ 1849.6 is sometimes reversed. It is also very sensitive to the presence of films on the window. This may partly explain the decay of the efficiency of the mercury arc with time, which has been recorded by several observers.[1] In the quartz arc, however, where the whole system remains very hot, the formation of films is probably not the important factor. The observed effect may well be due in part to a change in the quartz itself under the action of the ultra-violet light similar to that which I have observed with fluorite. With this substance, long exposure in the Schumann region will turn transparent, colorless fluorite into an opaque modification of a purple tinge. Dr. Nutting has observed the same change of color with glass.

It is interesting to note that Steubing's lines were obtained both with the quartz lamp and with the arc at low pressures.

The third column in the table gives the wave-lengths of the arc lines in vacuum. The line at λ 1849.6 was measured by direct

[1] *Comptes rendus*, **155**, 141, 1912.

comparison with the spark lines of aluminum. The other wave-lengths were found by comparison with a shifted spectrum of iron. Owing to the broad character of the arc lines and to the strong background due to scattered light, the setting error is larger than in the case of spark lines and the results may not be so accurate. Estimating the intensities which are given in the fourth column is a difficult matter. The line at λ 1849.6 is vastly stronger than any of the others and I have indicated the fact by the expression >100. The line marked "8" appears to have the same intensity as a line marked "8" in the hydrogen spectrum lying in the same region. It is obvious that these numbers give but a rough measure of the relative intensities.

The small number of lines in the arc spectrum is striking and may be partly attributed to the absorption of the mercury vapor within the lamp itself. Hughes predicted this scarcity of radiation from his photo-electric observations, but his estimate of the position of these lines turns out to be quite inaccurate even when his computation is corrected to agree with his recent experiments.

It is to be regretted that experimental difficulties made it impossible to verify Paschen's predictions beyond λ 1300. Radiation of wave-lengths shorter than λ 1300 must be strongly absorbed by fluorite, if not by the vapor in the lamp itself, and, therefore, only lines of the strongest character can be detected in this region.

I hope the results of this investigation may be of some value, first, because two of the important lines predicted by Paschen have been discovered; second, because of the information which has been obtained on the subject of the spark spectrum; third, because the nature of the arc spectrum has been determined and thus the radiation from the quartz mercury lamp has been definitely settled.

JEFFERSON LABORATORY
CAMBRIDGE

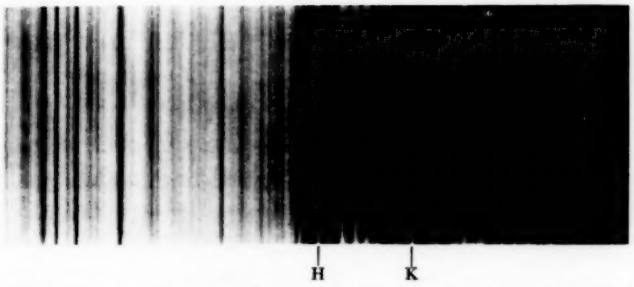
Minor Contributions and Notes

ON THE REVERSAL OF THE CALCIUM LINES H AND K IN STELLAR SPECTRA

During the year 1900 G. Eberhard and H. Ludendorff at the request of H. C. Vogel made photographs of stellar spectra with the small spectrograph D, which were intended particularly for the study of the ultra-violet. In the visible part of the spectrum these plates were very strongly overexposed. Eberhard found on such a plate of *Arcturus* that the K line of calcium shows a sharp reversal, that is, a calcium emission takes place in the middle of the absorption line just as one observes it in a disturbed region of the solar surface. As the absorption in the broad calcium line is very strong and the emission rather weak, the reversal is visible only on a plate in which the continuous spectrum in the neighborhood of the H and K lines is strongly overexposed. Further observations of this same phenomenon have recently been made by Schwarzschild with an objective-prism. On the plates of the *Hyades*, referred to in the preceding paper, taken with the U.V. Zeiss triplet and U.V. objective-prism (150 mm aperture, 1500 mm focus, dispersion 10 mm between H_γ and K) are found strongly exposed spectra of *Aldebaran*, which likewise show reversal of the K line and, although more weakly, a reversal of H. Similar exposures on α *Aurigae* and α *Ursae Majoris* failed to show reversal; neither did exposures on α *Orionis* and α *Cassiopeiae* show it. In the case of the last two it is perhaps true that the exposure was not sufficiently strong. An exposure of one hour on β *Geminorum* with the objective-prism showed no reversal, but on the same plate a neighboring star, σ *Geminorum*, shows extraordinarily bright, sharp emission lines in the middle of the absorption lines.

Although a mere examination of the spectra makes it very probable that one is dealing with a real emission phenomenon and not merely with a vacant space in the continuous spectrum, it

seems advisable to investigate the relation of this line to calcium by a determination of its wave-length. In the following the results of such measures are given as differences in radial velocity. The reversals were compared with 7 to 10 of the neighboring star lines due to *Fe* and *Al*.



Spectrum of σ *Geminorum* taken with the Zeiss triplet and U.V. objective-prism, 1913, March 9. Enlarged and broadened about 12 times.

α *Boötis*

Great Refractor. Spectrograph III with short camera. Slit-width 0.10 mm, width of spectrum 0.05 mm. February 23, 1913. Exposure 30^m

H -3 km/sec., K $+6$ km/sec.

α *Boötis*

Zeiss triplet with objective-prism. Width of spectrum 0.2 mm. April 23, 1912. Two exposures of 14^m and 7^m. The mean of both taken.

H $+5$ km/sec., K $+5$ km/sec.

α *Tauri*

The same apparatus. January 14, 1912. One exposure of 30^m. January 1, 1913. Two exposures of about 30^m each. Mean of all three.

H -3 km/sec., K $+3$ km/sec.

σ *Geminorum*

The same apparatus. March 9, 1913 and March 12, 1913. Each exposure about 60^m. Mean of the two.

H -6 km/sec., K -7 km/sec.

Since in the vicinity of H and K 1 μ equals 1.2 km/sec. with the spectrograph and 2.6 km/sec. with the objective-prism, the above numbers probably only mean that the wave-lengths of the reversals agree within errors of measurement with the wave-lengths of the H and K lines. The brightness of the H and K reversals in the case of σ *Geminorum* is so great that their blackening of the plate equals that of the brightest part of the continuous spectrum between H and K. The width of the emission line amounts to about one Å.U. on the plate taken March 12. On account of the unsteadiness of the air and the pointing error this is not a measure of the width of the line, which presumably is small, but only indicates the strength of the radiation. In the case of *Aldebaran* it is shown, by a comparison of its spectrum with that of other stars in the *Hyades* photographed on the same plate, that the brightness of the reversal with an equal width is relative to the continuous spectrum about three magnitudes weaker than that of σ *Geminorum*. *Arcturus* shows a slightly stronger reversal than *Aldebaran*. K is stronger than H in all the three stars. In a high-dispersion spectrum of the sun a very fine, weak reversal of the K line is seen all over its surface. In a detached region of disturbance the reversal reaches a strength as great as in σ *Geminorum*. In order to get an idea of the average strength of emission of the sun compared with that of the star, the spectrum of diffuse skylight was photographed with the same dispersion on March 14, 1913. The spectrum of diffused skylight of course corresponds to the average spectrum of the sun. This plate showed no sign of reversal of the H and K lines. From this we conclude that the emission is much stronger in these stars than in the sun.

The following general remarks may be added. Reversals of lines in stellar spectra are not rare. The reversals found here are interesting in that they take place in stars whose spectra are similar to that of the sun and therefore more comprehensible to us. The same kind of eruptive activity that appears in sun-spots, flocculi, and prominences, we probably also have to deal with in *Arcturus* and *Aldebaran* and in a very greatly magnified scale in σ *Geminorum*. This agrees with Adams'[1] result for *Arcturus*, viz.,

[1] *Astrophysical Journal*, **24**, 69, 1906.

its absorption lines show the transition from the normal solar spectrum to the spot spectrum.

Two problems arise in this connection. It remains to be shown whether the emission lines of the star have a possible variation in intensity analogous to the sun-spot period. It is known as a result of the investigations of Deslandres[1] and of St. John[2] that the centers of the calcium emission of the sun have a radial outward motion of 1 km per second. A more accurate determination of the wave-lengths of the calcium emission of the stars should prove whether or not a greater total intensity of emission is accompanied by a greater velocity of ascent.

G. EBERHARD
K. SCHWARZSCHILD

ON THE RADIAL VELOCITY OF *63 TAURI*

For the determination of radial velocities by means of the objective-prism E. C. Pickering has proposed the method of making two exposures of a star field on the same plate, reversing the prism 180° between the exposures. The distances of corresponding lines in the two spectra of each star then depend upon its radial velocity, and the measurement of these distances makes it possible to obtain the radial velocity after the lines have been identified, and the necessary reduction constants have been applied.

Instead of determining the radial velocity itself, the problem may be limited to finding variation in radial velocity by means of a comparison of two plates with such double exposures. Let the linear distance for a star on the first plate equal s, on a second equal s', then the difference $s'-s$ at once measures the change in radial velocity of the star if the observing conditions were absolutely identical. The inevitable changes in these conditions have the result that to the effect of the radial velocity is added that of a linear function of the rectangular co-ordinates x, y of the star. For the measured distances $s'-s$ we have equations of the form $s'-s=\alpha+\beta x+\gamma y+\Delta(s'-s)$, in which α, β, γ are plate-constants depending upon the observing conditions and in which the only remaining

[1] *Comptes rendus*, 1905, 381. [2] *Astrophysical Journal*, **32**, 36, 1910.

term, $\Delta(s'-s)$, represents the effect of the variable radial velocity. The three constants α, β, γ may be determined from stars whose radial velocities are constant. By deriving the remainder $\Delta(s'-s)$ for the other stars, the change in radial velocity may be obtained from it without further trouble. In case not only linear but also quadratic terms should have appreciable values in the expression for $(s'-s)$, these may easily be got by direct computation, as will be shown elsewhere.

The measurement of the difference $s'-s$ is made in the simplest manner by optically juxtaposing the spectra from the two plates by reflection. This I accomplished by means of the Zeiss stereo-comparator after making a small alteration in its "Blink" microscope. The advantages are the same as in the method of measuring spectrograms with the Hartmann spectrocomparator.

Groups of lines of both spectra may be brought to view, identification of the lines is superfluous, and since the comparison is between two spectra of one and the same star it is possible to employ also the extremely weak lines which on a single plate one would hardly dare to use.

Whether this method will come to be of any importance, in the search for variable radial velocities, compared with the performance of the great telescope and slit-spectrograph is a question about which I hardly dare to express an opinion. Meanwhile I can communicate a result which I have obtained by means of it. Three double exposures on the *Hyades* made on January 10, 1912, January 1 and January 4, 1913 were compared in the manner described above. The instrument with which they were taken was the Zeiss triplet of 150 mm aperture and 1500 mm focal length, combined with an objective-prism having a dispersion of 10 mm from H_γ to K. The comparison yielded for the star *63 Tauri* (1900.0, $4^h17.7^m$, $+16°33'$) the following deviation from the mean value (unknown): + 5 km/sec., −37 km/sec., + 33 km/sec.

As the star has good lines these numbers exceed by a great deal the errors of observation, and it is safe to conclude that this star has a variable radial velocity of short period and of large amplitude. The magnitude of this star is given in *H.R.* as 5.68, its type as A2. In *P.D.* its magnitude is 5.93. Its spectrum may be obtained in

60^m with the large refractor and spectograph III, since the spectrograph has been equipped with a short camera having the new chromat[1] lens of 180 mm focal length and ratio 1:4.5. Plates taken with the spectrograph confirmed the variation. The amplitude of variation may be 70–80 km/sec., the period in the neighborhood of eight days. A more exact investigation is in progress.

Because of its proper motion and evidently also because of its mean radial velocity, this star belongs physically to the *Hyades*. According to Hertzsprung[2] it is abnormal in that it has a K line which is weaker compared with the hydrogen lines than the K line of other stars of equal magnitude belonging to the *Hyades* system.

K. SCHWARZSCHILD

THE CONDITION FOR ACHROMATISM AND ANASTIGMATISM IN A SYMMETRICAL PHOTOGRAPHIC DOUBLET CONSISTING OF FOUR SEPARATED THIN LENSES

In applying the elegant methods for discussing the aberrations of systems of lenses, published by Taylor in his *A System of Applied Optics* to that type of photographic objective, such as the Goerz "Celor" and "Syntor," and the Steinheil "Unofocal," which consists of four separated thin[3] lenses arranged in the form of a symmetrical doublet, I discovered a simple relation which must be satisfied in order that the objective may be as free as possible from the errors of achromatism and astigmatism. As I have never run across this law in print, it has occurred to me that it may be new to others and that it is of sufficient interest to warrant its publication. This condition stated in general terms is as follows: *The separation between the positive and negative lens minus the focal length of the negative lens must equal the focal length of the positive lens multiplied by the square root of the ratio of the indices of refraction divided by the ratio of the dispersive powers.*

[1] *Sitzungsber. d. Berl. Akad. d. Wiss.*, p. 1220, 1912.

[2] *Publ. des Astrophys. Observatoriums zu Potsdam*, No. 63, p. 35.

[3] As actually constructed, these lenses are not *mathematically* thin and hence the following law does not apply with full rigor.

The proof of this proposition is as follows: on p. 285 Taylor gives for the achromatism of n separated thin lenses the following expression:

$$\Delta\frac{1}{v_n}=\frac{1}{f_1}\frac{\Delta\mu_1}{\mu_1-1}\left(\frac{v_1v_2\ \ldots\ v_{n-1}}{u_2u_3\ \ldots\ u_n}\right)^2+\frac{1}{f_2}\frac{\Delta\mu_2}{\mu_2-1}\left(\frac{v_2\ \ldots\ v_{n-1}}{u_3\ \ldots\ u_n}\right)^2\ \ldots\ +\ \frac{1}{f_n}\frac{\Delta\mu_n}{\mu_n-1}. \tag{1}$$

In the above expression the v's denote the distances of the image formed by any lens from the lens and the u's the distances of the objects, the f's being the focal lengths. In the special case under consideration let the focal lengths be f, $-kf$, $-kf$, and f and the separations be t, $2d$, and t, d being, therefore, the distance of the stop from the second lens of the system. We shall also have $\mu_1=\mu_4$ and $\mu_2=\mu_3$. Let us put $a=\frac{\mu_1-1}{\Delta\mu_1}\cdot\frac{\Delta\mu_2}{\mu_2-1}$. Introducing these values into (1) we have at once:

$$\Delta\frac{1}{v_4}=\frac{1}{f}\frac{\Delta\mu_1}{\mu_1-1}\left\{\left(\frac{v_1v_2v_3}{u_2u_3u_4}\right)^2-\frac{a}{k}\left[\left(\frac{v_2v_3}{u_3u_4}\right)^2+\left(\frac{v_3}{u_4}\right)^2\right]+1\right\}=\frac{1}{f}\cdot\frac{\Delta\mu_1}{\mu_1-1}G. \tag{2}$$

In order to evaluate G we proceed as follows: from the ordinary law of conjugate focii we have

$$\frac{1}{u_2}=\frac{1}{v_2}+\frac{1}{kf}$$

or

$$u_2=\frac{kfv_2}{kf+v_2}.$$

But evidently we have

$$u_2=v_1-t$$

whence we obtain

$$v_1=\frac{kfv_2+kft+v_2t}{kf+v_2}$$

and, therefore,

$$\frac{v_1}{u_2}=\frac{v_2(kf+t)+kft}{kfv_2}.$$

In the same way it may be shown that

$$\frac{v_3}{u_4}=\frac{kfu_3}{(kf+t)u_3-kft}.$$

From these it readily follows, after a slight reduction, that

$$\frac{v_1v_2v_3}{u_2u_3u_4}=\frac{v_2(kf+t)+kft}{u_3(kf+t)-kft}$$

$$\frac{v_2v_3}{u_3u_4}=\frac{kfv_2}{u_3(kf+t)-kft}.$$

Substitute these values into the expression for G and we have:

$$[v_2(kf+t)+kft]^2-\frac{a}{k}[k^2f^2v_2^2+k^2f^2u_3^2]+[u_3(kf+t)-kft]^2=AG$$

where

$$A=[u_3(kf+t)-kft]^2. \tag{3}$$

Let $y=v_2-d$. We then have $v_2=y+d$ and, since $u_3=v_2-2d$, $u_3=y-d$. Introducing these values we have, after reduction:

$$[v_2(kf+t)+kft]^2=(kf+t)^2(y^2+2dy+d^2)+2kft(kf+t)(y+d)+k^2f^2t^2$$

$$[u_3(kf+t)-kft]^2=(kf+t^2)(y^2-2dy+d^2)-2kft(kf+t)(y-d)+k^2f^2t^2$$

$$[v_2(kf+t)+kft]^2+[u_3(kf+t)-kft]^2=$$

$$2(kf+t)^2(y^2+d^2)+4akft(kf+t)d+2k^2f^2t^2.$$

In the same way we have:

$$k^2f^2v_2^2+k^2f^2u_3^2=2k^2f^2[y^2+d^2].$$

Introducing these values into equation (3) we find, after a slight reduction,

$$y^2[(kf+t)^2-akf^2]+[d(kf+t)+kft]^2-akd^2f^2=\tfrac{1}{2}AG.$$

This gives finally

$$\Delta\frac{I}{v_4}=\frac{2}{f}\cdot\frac{\Delta\mu_1}{\mu_1-1}\left\{\frac{y^2[(kf+t)^2-akf^2]+[d(kf+t)+kft]^2-akd^2f^2}{[(y-d)(kf+t)-kft]^2}\right\}.$$

Let us divide both terms of this fraction by f^4:

$$\Delta\frac{I}{v_4}=\frac{2}{f}\cdot\frac{\Delta\mu_1}{\mu_1-1}\left\{\frac{\frac{y^2}{f^2}\left[\left(k+\frac{t}{f}\right)^2-ak\right]+\left[\frac{d}{f}\left(k+\frac{t}{f}\right)+k\frac{t}{f}\right]^2-ak\left(\frac{d}{f}\right)^2}{\left[\left(\frac{y-d}{f}\right)\left(k+\frac{t}{f}\right)-k\frac{t}{f}\right]^2}\right\}.$$

Now in this type of objective the separations are kept as small as possible and hence $\frac{t}{f}$ and $\frac{d}{f}$ are small in comparison to $\frac{y}{f}$ since y is the distance that the focus of the first combination is back of the stop. The second and third terms in the numerator are then considerably smaller than the first. Furthermore, it is to be noted that the first term of the numerator is the only one that varies with the distance of the object from the lens; hence it follows that, everything considered, the best achromatism will be reached in this type of lens if the first term be made zero, which gives us

$$(kf+t)^2-akf^2=0$$

or

$$kf+t=f\sqrt{ak}\,. \qquad (4)$$

Now in order that the excentric corrections due to the stop may be exactly three times in the primary plane what they are in the secondary plane and hence be simultaneously made equal to zero, the well known Petzval condition must be satisfied, namely,

$$\Sigma\frac{1}{\mu f}=0=2\left[\frac{1}{\mu f}-\frac{1}{k\mu_2 f}\right]$$

whence

$$k\mu_2-\mu_1=0$$

or

$$k=\frac{\mu_1}{\mu_2}\,.$$

Hence in (4) k is the ratio of the indices of refraction and, as was shown above, a is the reciprocal of the dispersive powers.

H. C. Lord

Emerson McMillin Observatory
Columbus, Ohio

THE INTERNATIONAL UNION FOR CO-OPERATION IN SOLAR RESEARCH

The fifth conference of the Union was held in Bonn, Germany, from July 30 to August 5, 1913. On the first day of the conference the sun appeared for the first time in several weeks, and it continued to shine until the Union adjourned to meet in Rome in 1916.

The welcome extended by the local hosts to the visiting scientists was no less warm than that bestowed upon them by the object of their research, and the delightful social features arranged for their entertainment will always be remembered with the greatest of pleasure.

On the evening of July 30 the city of Bonn gave a banquet in their honor. During the afternoon and evening of August 1, Professor and Mrs. Küstner gave a reception and tea at the Bonn Observatory. On the next day the city of Cologne entertained the Union, the program including a tour of the city, visits to the cathedral and art gallery, and a banquet in the Gürzenich. Sunday, August 2, was devoted to excursions. A part of the company took an all-day tramp through the Siebengebirge; the rest, to the number of about 60, went on an automobile tour through the Eifelgebirge to Cochem, down the valley of the Mosel to Coblenz, and back along the Rhine to Bonn, a total distance of 130 miles. The sixteen machines used were kindly loaned by residents of Bonn.

Monday evening, August 4, was devoted to a *conversazione* in the Physical Institute. An opportunity was given for an informal exchange of ideas upon a great variety of subjects, not necessarily associated with solar research. In the various rooms there were exhibits by Chrètien, Donitch, Goos, Hemsalech, Hertzsprung, Leybold, W. Michelson, Pflüger, E. C. Pickering, Russell, Slocum, Störmer, Stratton, de Watteville, and Wolf. This proved to be a most enjoyable and profitable occasion and may become a permanent feature of the Union conferences.

Tuesday afternoon and evening Professor Kayser gave a party on the Rhine. This included an excursion up the river to Rolandseck, visit to the oldest ruin on the Rhine, and dinner

on board the boat. Upon the return of the steamer to Bonn, many of the party took a night train to Hamburg to be present at the first meeting of the Astronomische Gesellschaft the next morning.

There were five formal scientific sessions of the conference, presided over respectively by Professors Kayser, Küstner, Schwarzschild, Runge, and Pringsheim. Professors Konen, Fowler, Hemsalech, and Count de la Baume Pluvinel acted as secretaries.

The sessions were devoted chiefly to the consideration of the reports of the various committees, appointed to promote co-operation in solar research and allied investigations. Nine committees reported.

In the absence of Professor Schuster, Professor Turner acted as chairman of the Executive Committee and presented its report. Professor Turner also presented the report of the Computing Bureau. The work of the Bureau during the past three years has been confined chiefly to the measurement of the areas of sun-spots from the observations made by Professor Peters. In the near future the measurement of spectroheliograms will be undertaken at Cambridge, and the results communicated to the Union at its next meeting.

The report of the Committee on Solar Radiation was presented by Mr. Abbot.

The report considered (*a*) The Variability of the Sun: (*b*) Atmospheric Transmission; (*c*) Light of the Sky; (*d*) Nocturnal Radiation; (*e*) The Solar Constant of Radiation; (*f*) Distribution of Radiation over the Sun's Disk.

The following summary is appended to the report:

> In view of what has been reported the committee feels that the period which has elapsed since the last meeting of the Solar Union has been very productive in results on the quantity and effects of the solar radiation. In brief: The standard scale of radiation seems to be closely fixed; the mean value of the solar constant of radiation is apparently well determined; the variation of the quantity of solar radiation in association with the sun-spot numbers appears to be strongly indicated; a short irregular periodicity of the sun seems to be established; and much valuable work in connection with the distribution of radiation at different parts of the earth's surface, the

measurement of the light of the sky and of nocturnal radiation and on the connection between solar radiation and meteorology has made substantial progress.

In connection with this report a communication from Mr. Evershed was read, suggesting the possibility of studying the variation of solar radiation by comparing the light of the moon and planets with starlight.

The report of the Committee on Standards of Wave-Lengths was presented by Professor Kayser.

The accuracy of interferometer measures has been confirmed, and it has been decided that for the measurement of the tertiary standards either plane or concave gratings may be used.

It has been found that many lines vary according to the length of the arc, the part of the arc used, and the strength of the current. These variations are similar to those obtained as pressure-effects for various groups of lines at Mount Wilson. It seems necessary therefore, to define the source of light, and for future co-operation in the measurement of the tertiary standards, the committee made the following recommendations:

1. That the length of the arc should be 6 mm.

2. That the current should be 6 amperes for wave-lengths greater than 4000 Å units, and 4 amperes or less for the shorter wave-lengths.

3. That continuous current should be employed, the positive pole be above the negative, the p.d. be 2200 volts, and that the iron rods be of 7 mm diameter.

4. That the source of light examined should be a length of 2 mm at the center of the arc.

5. That only the Mount Wilson groups *a*, *b*, *c*, *d* be employed, and that for *c* and *d* the slit should be at right angles to the arc and the current should be reversed several times during the exposure.

The report of the Committee on the Classification of Stellar Spectra was presented by Professor Schlesinger.

Soon after the 1910 meeting of the Union the committee sent out the following questions:

1. At the meeting of the Committee held on Mount Wilson, there seemed to be a practically unanimous opinion that the Draper classification is the most useful that has thus far been proposed. Do you concur in this opinion? If not, what system do you prefer?

2. In any case, what objections to the Draper classification have come to your notice and what modifications do you suggest?

The report of the committee consisted chiefly in a discussion of the answers to these questions. These answers have already been published in the *Astrophysical Journal*, **33**, 260, April 1911.

The following resolutions were proposed:

1. That the Committee on the Classification of Stellar Spectra be asked to secure by co-operation the material necessary for the establishment of a system that can be recommended for permanent and universal adoption; and

2. That, pending the establishment of such a system, the use of the Draper classification be recommended in the form described in Vol. **56**, p. 66, of the *Annals of the Harvard College Observatory;* except that hereafter, in accurate classification, a zero be added to letters not followed by other numerals and that the absence of any numeral be taken to indicate only a rough classification.

The report of the Committee on the Determination of the Solar Rotation of Means of the Displacement of Lines was presented by Dr. Plaskett.

The report described the results obtained by Story and Wilson, Hubrecht, Plaskett and DeLury, and emphasized especially the discrepancies in the measures. Different observers have obtained results differing systematically by as much as 10 per cent from one another. The following resolutions were, therefore, proposed:

1. It is highly desirable to trace to their source the systematic differences that are found in the value of the solar rotation by different observers, and this investigation should take precedence of a continuation of the program adopted in 1910.

2. For this purpose study should be directed to determining the velocity at the solar equator by as many different methods as possible, and it is recommended that the ten lines chosen in 1910 in the region $\lambda 4220$–$\lambda 4280$ be used for this purpose.

3. That investigation should also be made into the personal differences that are found in measures of the same plates by different observers.

The report of the Committee on the Investigation of the Spectra of Sun-Spots was presented by Professor Fowler.

On account of the quiescent condition of the sun during the past three years the committee had very little fresh information of value. The report was devoted chiefly to the details of co-operation. The following recommendations were made:

1. That the observers should be requested to continue systematic visual observations of the umbral spectrum for at least another three years, so as to

complete a sun-spot cycle. Such observations may conveniently be restricted to a selected list of lines which has been approved by the committee.

2. That the observers be recommended to continue observations of H_α and D_3 in the neighborhood of spots, and to give attention to other phenomena mentioned in the committee's further suggestions to observers.

3. That in consideration of promises of co-operation already received, the secretary be authorized to communicate with observers possessing equipment for photographic investigations of spot-spectra with a view to organizing co-operation in the preparation of a catalogue of affected lines and possibly in other investigations.

The report of the Committee for the Organization of Eclipse Observations was presented by Count de la Baume Pluvinel. The report contained a brief description of the results obtained in recent total and annular eclipses and mentioned some plans for co-operation in the observation of the eclipse of 1914. Astronomers intending to locate in Russia should notify Dr. N. Donitch, 25 Moïka, St. Petersburg, and state the weight of their apparatus.

The report of the Committee on Work with the Spectroheliograph was presented by Professor Slocum.

As a part of the report accounts of the progress of work at the various observatories were given by Riccò, St. John, Slocum, Chrètien, Donitch, Ascarza, Butler, and Kempf.

The following recommendations of the committee were adopted:

1. That a systematic comparison be made of the results from different types of spectroheliograms, as given by instruments of varied form and dispersive power, utilizing as many observations as possible.

2. That the observatories of Coimbra, Nice, and Starya Doubossary, Bessarabia, South Russia, having now acquired spectroheliographs, should be added to our list, and that their directors be invited to send delegates to our meetings.

3. That spectroheliographs of high dispersion capable of recording the details of the higher atmosphere with the K_3 or H_α lines, combined with image-forming apparatus of long focus, should be installed wherever possible.

4. That the "*spectro-enregistreur des vitesses*" for the *radial* velocities be recommended especially for use with the hydrogen and calcium lines.

5. That the Union express its hearty gratification with regard to the reported possibility that the memorial to Secchi may take the form of a tower telescope in the vicinity of Reggio (Emilia), for the general study of the solar atmosphere, and that this undertaking be supported as a desirable addition to the equipment for solar observations in northern latitudes.

6. That the title of the committee be changed to the more general name "The Committee on Solar Atmosphere," so as to include and to unify all the observations on the solar atmosphere, visual and photographic, except those associated with eclipses.

It is suggested that this committee may then be composed of two sub-committees: the *first* to be devoted to visual observations of prominences and related phenomena; the *second*, to photography of the forms, and determination of the velocities manifested in the solar atmosphere, both of the disk and limb.

The committee wishes to record that it is recognized that for many years there has been in existence an active and efficient organization for co-operation in the visual observations of prominences, and it is hoped that by its inclusion within this Committee on Solar Atmosphere, this and other similar bodies may be induced to join in the work of the Union.

After the adoption of the last recommendation, the sub-committee for the visual observation of prominences was appointed with Professor Riccò as chairman and Father Cortie as secretary. This committee held a meeting and voted to present the following recommendations, all of which were adopted:

1. That observers be recommended to observe not only the height of prominences, but the arc they cover on the sun's limb and to express their results for each prominence as profile areas.

2. That an area covered by an arc one degree (of the sun's limb) in length and one second (of arc of the celestial sphere) in height be adopted as the conventional unit of profile area; and that it be called the prominence unit.

3. That the limiting height for statistics of frequency be 30″.

4. That the position angles round the sun's limb be recorded in the direction N.E.S.W. from the apparent north point.

5. That, if possible, the height of the chromosphere be measured at every 45°, and at other points where it is notably above or below its normal level.

6. That prominences be classified into very high, high, moderately high, faint, very faint.

In accordance with the custom of the Union there were no formal papers, but the following special reports were presented:

Buisson, Standards of Wave-Lengths.
Deslandres, Solar Work at Meudon.
Riccò, Statistics of Solar Prominences.
Julius, Refraction of Light in Passing through Whirling Gases.
Julius, Interpretation of Sun-Spots.
St. John, Radial Motion in Sun-Spots.
St. John, Rotation of the Sun as Determined by Different Lines.

Hale (read by St. John), General Magnetic Field of the Sun.

Hemsalech, Influence of a Transverse Magnetic Field on the Aspect and Spectrum of a Calcium Spark in Hydrogen.

Störmer, Observations of the Aurora.

Störmer, Interpretation of Solar Phenomena.

Duffield, Conditions in Australia in Regard to a New Government Solar Observatory.

The following are the committees of the Union as announced at the Bonn meeting:

1. On Standards of Wave-Lengths:
   Kayser (*chairman*), Ames, Burns, Buisson, Fabry, Goos, Michelson, Paschen, Perot, St. John.
2. On the Measurement of Solar Radiation:
   Violle (*chairman*), Abbot (*secretary*), Callendar, Chistoni, Evershed, Julius, Pringsheim, Schuster.
3. On Solar Atmosphere:
   a) Visual (prominences):
   Riccò (*chairman*), Cortie (*secretary*), Ascarza, Butler, Chevalier, Deslandres, Evershed, Fenyi, Fowler, Jiminez, Kempf, Tringali, Whitelow.
   b) Photographic (using spectroheliograph, *enregistreur des vitesses*, spectrograph):
   Hale (*chairman*), Chrètien, Cirera, Costa Lobo, Deslandres, Donitch, Evershed, Frost, Iñiguez, Kempf, W. J. Lockyer, Newall, Riccò, Slocum, St. John.
4. On the Investigation of the Spectra of Sun-Spots:
   Newall (*chairman*), Fowler (*secretary*), Adams, Belopolsky, Cortie, Deslandres, Dyson, Evershed, Fox, Hale, N. Lockyer, W. Mitchell, Plaskett, Wolfer.
5. On the Organization of Eclipse Observations:
   N. Lockyer (*chairman*), de la Baume Pluvinel (*secretary*), Blumbach, Campbell, Cirera, Donitch, Fowler, Hills, Kempf, Riccò, Strömgren, Turner.
6. On the Determination of the Solar Rotation by Means of the Displacement of Lines:
   Dunèr (*chairman*), Adams (*secretary*), Belopolsky, Deslandres, Dyson, Halm, Hubrecht, Newall, Perot, Plaskett, Samson, Schlesinger.
7. On the Classification of Stellar Spectra:
   E. C. Pickering, (*chairman*), Schlesinger (*secretary*), Adams, Belopolsky, Campbell, Miss Cannon, Fowler, Frost, Hale, Hamy, Hartmann, Hertzsprung, Iñiguez, Kapteyn, Küstner, Newall, Plaskett, Russell, Schwarzschild.

F. SLOCUM

The following astronomers and physicists were present:

C. G. Abbot, Smithsonian Astrophysical Observatory, Washington, D.C.
Sir W. de W. Abney, Atherstone, England.
Miss L. B. Allen, Whitin Observatory, Wellesley, Mass.
J. S. Ames, Johns Hopkins University, Baltimore, Md.
V. F. Ascarza, Observatorio, Madrid, Spain.
O. Backlund, Observatoire de Poulkova, Poulkova, Russia.
S. I. Bailey, Harvard College Observatory, Cambridge, Mass.
B. Baillund, Observatoire de Paris, Paris, France.
J. Baillund, Observatoire de Paris, Paris, France.
A. Belopolsky, Observatoire de Poulkova, Poulkova, Russia.
F. J. Blumbach, St. Petersburg, Russia.
J. Bosler, Observatoire de Meudon, Meudon, France.
H. Buisson, Université d'Aix-Marseille, Marseille, France.
K. Burns, Bureau of Standards, Washington, D.C.
C. P. Butler, Solar Physics Observatory, Cambridge, England.
W. W. Campbell, Lick Observatory, Mount Hamilton, Cal.
Miss A. J. Cannon, Harvard College Observatory, Cambridge, Mass.
H. Chrètien, Observatoire de Nice, Nice, France.
A. L. Cortie, S. J., Stonyhurst College Observatory, Lancashire, England.
A. Cotton, École Normale Supérieure, Paris, France.
F. Croze, Paris, France.
H. Deslandres, Observatoire de Meudon, Meudon, France.
N. Donitch, Observatoire de l'Université, St. Petersburg, Russia.
C. L. Doolittle, Flower Observatory, Philadelphia, Pa.
W. G. Duffield, London, England.
F. W. Dyson, Royal Observatory, Greenwich, England.
G. Eberhard, Astrophysikalisches Observatorium, Potsdam, Germany.
A. S. Eddington, Cambridge University, Cambridge, England.
P. Eversheim, Universität, Bonn, Germany.
E. Fayet, Observatoire de Nice, Nice, France.
A. Fowler, Imperial College of Science and Technology, South Kensington, London, England.
R. Furuhjelm, Astronomisches Observatorium, Helsingfors, Finland.
Prince B. Galitzin, Observatoire Physique Central Nicolas, St. Petersburg, Russia.
H. Giebeler, Universität, Bonn, Germany.
C. H. Gingrich, Carleton College, Northfield, Minn.
L. C. Glaser, Berlin, Germany.
F. Goos, Physikalisches Staatslaboratorium, Hamburg, Germany.
L. Grebe, Universität, Bonn, Germany.
J. G. Hagen, S. J., Vatican Observatory, Rome, Italy.
J. Hartmann, Königliche Sternwarte, Göttingen, Germany.

K. Haussmann, Technische Hochschule, Aachen, Germany.
D. E. A. Hemsalech, Paris, France.
E. Hertzsprung, Astrophysikalisches Observatorium, Potsdam, Germany.
J. v. Hepperger, K. K. Sternwarte, Vienna, Austria.
Major E. H. Hills, London, England.
O. Holz, Universität, Bonn, Germany.
J. Hopmann.
J. B. Hubrecht, Knutsford, England.
L. Janicki, Physikalische Technische Reichsanstalt.
P. Jiménez, Observatorio Astronomico, Madrid, Spain.
W. H. Julius, University, Utrecht, Holland.
H. Kayser, Universität, Bonn, Germany.
P. Kempf, Astrophysikalisches Observatorium, Potsdam, Germany.
H. Knox-Shaw, Helwan Observatory, Helwan, Egypt.
H. Konen, Physikalisches Institut, Münster, Germany.
F. Küstner, Königliche Sternwarte, Bonn, Germany.
H. Ludendorff, Astrophysikalisches Observatorium, Potsdam, Germany.
C. Mönnichmeyer, Königliche Sternwarte, Bonn, Germany.
W. Michelson, Moscow, Russia.
E. L. Nichols, Cornell University, Ithaca, N.Y.
J. W. Nicholson, Cambridge University, Cambridge, England.
F. Nierhoff, Universität, Bonn, Germany.
Baron v. d. Pahlen, Caputh bei Potsdam, Germany.
J. A. Parkhurst, Yerkes Observatory, Williams Bay, Wis.
F. Paschen, Universität, Tübingen, Germany.
A. Pflüger, Universität, Bonn, Germany.
E. C. Pickering, Harvard College Observatory, Cambridge, Mass.
V. d. Plaats, Utrecht, Holland.
J. S. Plaskett, Dominion Observatory, Ottawa, Canada.
H. C. Plummer, Dunsink, Royal Observatory, Dublin, Ireland.
Count A. de la Baume Pluvinel, Rue de la Baume, Paris, France.
E. Pringsheim, Universität, Breslau, Germany.
A. Riccò, Osservatorio Astrofisico, Catania, Sicily.
G. B. Rizzo, Regia Università, Messina, Italy.
C. Runge, Universität, Göttingen, Germany.
H. N. Russell, Halsted Observatory, Princeton, N.J.
C. E. St. John, Mount Wilson Solar Observatory, Pasadena, Cal.
P. Salet, Observatoire de Paris, Paris, France.
R. A. Sampson, Royal Observatory, Edinburgh, Scotland.
F. Schlesinger, Allegheny Observatory, Allegheny, Pa.
K. Schwarzschild, Astrophysikalisches Observatorium, Potsdam, Germany.
H. Shapley, Halsted Observatory, Princeton, N.J.
F. Slocum, Yerkes Observatory, Williams Bay, Wis.

J. Stebbins, University of Illinois, Urbana, Ill.
C. Störmer, Universitet, Christiania, Norway.
F. J. M. Stratton, Cambridge University, Cambridge, England.
E. Strömgren, Universitets-Observatoriet, Copenhagen, Denmark.
P. Stroobant, Observatoire Royal de Belgique, Uccle, Belgium.
H. H. Turner, University Observatory, Oxford, England.
G. Van Biesbroeck, Observatoire Royal de Belgique, Uccle, Belgium.
W. Voigt, Universität, Göttingen, Germany.
C. de Watteville, Paris, France.
E. Weiss, Vienna, Austria.
M. E. T. Whitelow, Southport, England.
Miss S. F. Whiting, Whitin Observatory, Wellesley, Mass.
M. Wolf, Grossherzogliche Sternwarte, Heidelberg, Germany.
W. Zurhellen, Königliche Sternwarte, Neu Babelsberg bei Berlin, Germany.